Transactions Papers

Downlink Resource Allocation in Multi-Carrier Systems: Frequency-Selective vs. Equal Power Allocation

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***Abstract*—This paper revisits equal power allocation from the viewpoint of asymptotic network utility maximization (NUM) problem in multi-carrier systems. It is a well-known fact that the equal power allocation is near optimal to the sum capacity maximization problem in high SNR (Signal-to-Noise Ratio) regime, i.e., optimal water-filling approximates to equal power allocation in that case. Due to this property together with its simplicity, the equal power allocation has been adopted in several researches, but its performance in other problems has not been clearly understood. We evaluate the suitability of equal power allocation in NUM problem which turns into various resource sharing policies according to utility functions. Namely, our conclusion is that in frequency selective channels, the equal power allocation is near optimal for efficiency-oriented resource sharing policy, but when fairness is emphasized, its performance is severely degraded and thus frequency-selective power allocation is necessary. For this, we develop a suboptimal subcarrier and frequency-selective power allocation algorithm for asymptotic NUM problem using the gradient-based scheduling theory and compare the performance of equal power allocation and the developed algorithm. Extensive simulation results are presented to verify our arguments.**

***Index Terms*—Dynamic subcarrier and power allocation, OFDMA downlink, network utility maximization, equal power allocation.**

I. Introduction

As the demand for high data rates over wireless networks increases, most of wireless standards such as IEEE 802.11/16/20 have adopted OFDM (orthogonal frequency division multiplexing) technology due to its high degree of flexibility and predominant performance over other air interface technologies. Accordingly, the resource allocation problem in multi-carrier systems has become a very important issue and attracted many researchers. The difficulty in dealing with such problems is that we have to simultaneously control multiple resources, e.g., subcarrier, power and bit, considering wireless channel conditions. This difficulty can be alleviated by fixing a particular resource and controlling other resources, at the cost of performance degradation. The equal power allocation, which uniformly distributes transmit power over all subcarriers, is the most commonly used alleviation because it separates all subcarriers and thus enables independent control of each subcarrier. More importantly, it is well-known that the equal power allocation for multiple access channels asymptotically maximizes the sum capacity maximization problem in high SNR (Signal-to-Noise Ratio) regime [1]. In other words, the performance gain achieved by frequency-selective optimal power allocation would be marginal compared to the equal power allocation. This was also observed in [2], [3] through simulations, and therefore the equal power allocation has been frequently utilized for the tractability of the resource allocation problem in multi-carrier setting [4], [5]. We revisit this equal power allocation from the viewpoint of asymptotic network utility maximization (NUM) problem.

The goal of asymptotic NUM is to maximize network utility over the long-term throughput region. This maximization can be achieved by gradient-based scheduler which selects the achievable data rate vector whose projection onto the gradient of network utility becomes maximal. The gradient-based scheduler can be regarded as finding the rate vector maximizing the sum of weighted rates where the weights are marginal utilities. It has been proved in several papers [6]–[8] that such a gradient-based scheduling maximizes network utility over the long-term throughput region. Thus, we can develop various resource sharing policies by changing the utility functions in the NUM problem. For example, if we use logarithmic utility functions, the gradient-based scheduler

Manuscript received December 25, 2006; revised August 5, 2007 and December 30, 2007; accepted August 11, 2008. The associate editor coordinating the review of this paper and approving it for publication was M. Chiang. Some part of this work was presented at IEEE WoWMoM conference, June 2007. This research was supported by the MIC(Ministry of Information and Communication), Korea, under the ITRC(Information Technology Research Center) support program supervised by the IITA(Institute of Information Technology Advancement) (IITA-2007-C1090-0701-0037).

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Digital Object Identifier 10.1109/T-WC.2008.061110

becomes PF (Proportional Fair) scheduler [9].

In this paper, we investigate whether or not the equal power allocation is really suitable for various NUM problems, i.e., various resource sharing policies. We first develop a suboptimal subcarrier and power allocation algorithm for NUM problem and use this algorithm for investigating the suitability. Our conclusion is that in frequency selective channels, the equal power allocation is near optimal for efficiency-oriented resource sharing policy, but when fairness is emphasized, its performance is severely degraded and thus frequency-selective power allocation is necessary. The efficiency-oriented resource sharing policy tries to maximize or enhance the total throughput, and thus a user with good channel condition is highly likely to be selected over each subcarrier at every time slot. This turns the system into high SNR regime. The optimal power allocation becomes approximately equal to equal power allocation in this case. In contrast, all the users should be scheduled under fairness-oriented resource sharing policy even if some of them experience bad channel conditions. The optimal power allocation in this case can be significantly different from equal power allocation. This is why the performance of equal power allocation shows different characteristics according to resource sharing policies.

A. Related Work

In [10], Wong et al. solve the total transmit power minimization problem with rate and BER (bit error rate) requirement constraints and propose an adaptive subcarrier, bit and power allocation scheme in multiuser OFDM systems. Since their pioneering work, there have been several related works that take snap-shot approach for resource allocation in OFDM systems [3], [11]–[14]. Their objectives are to maximize total throughput or utility, or to minimize total transmit power subject to rate and power constraints. All these previous works take *snap-shot approach* in which the resource allocation is developed through an optimization problem defined for fixed channel gain. Thus, short-term performance optimization is of their main concern[1].

On the other hand, there has been an effort to optimize the asymptotic system performance. In [15], Huang et al. consider achieving asymptotic NUM in OFDM downlink by using the gradient-based scheduling theory [6]–[8]. The problem is in the form of weighted sum rate maximization involving subcarrier and power allocation at each time slot. They adopt the FDM (frequency division multiplexing) rate model from [16] to impose the convexity of the problem. In [5], a similar problem with the same SNR-rate model is defined for finding the asymptotic capacity region of FDMA (frequency division multiple access) systems. The optimal solution can be efficiently found through the model, but in general it does not provide feasible optimal solution for OFDMA (orthogonal frequency division multiple access) systems where at most one user can be allocated to each subcarrier. Accordingly, they propose suboptimal algorithms such as allocating uniform power over selected (possibly not all) subcarriers [5] or allocating optimal power for given subcarrier allocation [15].

Our work is in line with [15] in that we also consider asymptotic NUM problem using gradient-based scheduling. The objective of [15] is to address how to achieve asymptotic NUM point, i.e., to find an optimal (if possible) subcarrier and power allocation for NUM problem. Consequently, they mainly focus on the analysis of optimality properties of their formulation and the algorithm development based on the analysis. The performances of their optimal and suboptimal algorithms are demonstrated through simulations. In fact, this is what has been typically done in the OFDM literature, i.e., identify that the problem is difficult and thus propose a suboptimal algorithm, whose performance is verified later in the simulation. We also use asymptotic NUM problem, but the main focus of this paper is different from that of [15]. We are interested in investigating the suitability of equal power allocation for various resource sharing policies in terms of efficiency and fairness. To do this, we develop a suboptimal algorithm using gradient-based scheduling. Although we use gradient-based scheduling as in [15], we consider a different rate model so that the problem to be solved at each time slot is different from [15].

So far in the OFDM literature, it seems that the equal power allocation is a commonly accepted policy which can enjoy excellent performance with simplicity [2]–[5]. It is thus important to identify the performance of equal power allocation, but its performance has been verified mainly in sum capacity maximization problem, and therefore it needs to be studied more in various policies. We believe that our result can work as some practical guidelines such that if we want efficiency, we can enjoy both simplicity and (near) optimality by using equal power allocation, but otherwise, we need to consider using frequency-selective power allocation rather than equal power allocation.

The rest of the paper is organized as follows. In Section II, we describe the system model and formulate an optimization problem involving subcarrier and power allocation for each time slot. Section III provides some comments on the difficulties of optimal algorithm. In Section IV, we present a suboptimal algorithm which will be used for the comparison with equal power allocation, and provide some observations on equal power allocation. Our claims are verified through extensive simulations in Section V, and the paper is concluded in Section VI.

II. Model Description and Problem Formulation

Consider the downlink of a single OFDMA cell with N subcarriers and M users. We denote by $\mathcal{M}$ and $\mathcal{N}$ the set of all users in the system and the set of all subcarriers, i.e., $\mathcal{M} = \{1, 2, ..., M\}$ and $\mathcal{N} = \{1, 2, ..., N\}$, respectively. Time slot $[t-1, t)$, $t = 1, 2...$ is called "time slot t", so from time 0 to time t, we have t number of time slots. For every time slot t, at most one user can be served over each subcarrier. Let r_{ij}^t be the data rate of user i achievable over subcarrier j during time slot t. We use the Shannon bound for SNR-rate model, i.e., $r_{ij}^t = B/N \log_2 \left(1 + \eta_{ij}^t p_j^t\right)$ where B is the system bandwidth, and η_{ij}^t and p_j^t respectively denote the received signal-to-noise-ratio (SNR) per unit power and transmit power allocated to subcarrier j during time slot t. Here, the sum of transmit powers allocated to all subcarriers must not exceed

[1]It should be noted that [14] also addresses the long-term optimal resource allocation by means of time diversity.

the total transmit power $\bar{P}$, i.e., $\sum_{j\in\mathcal{N}} p_j^t \le \bar{P}$ for each time t.

Let $R_i(t)$ be the average throughput of user i up to time t. Then, $R_i(t)$ is given by

$$R_i(t) = \frac{\sum_{j\in\mathcal{N}} \sum_{\tau=1}^{t} r_{ij}^{\tau} x_{ij}^{\tau}}{t} \quad (1)$$

where x_{ij}^{τ} is the indicator function such that $x_{ij}^{\tau} = 1$ if user i is selected at time $\tau - 1$ to be served over subcarrier j in time slot τ and $x_{ij}^{\tau} = 0$ otherwise. At most one user can be assigned to each subcarrier at each time slot, but there is no constraint on the number of subcarriers that can be allocated to a user. Hence, the indicator function must satisfy $\sum_{i\in\mathcal{M}} x_{ij}^t \le 1, \forall j$ for all time t. Each user $i \in M$ is associated with the utility function $U_i(R_i(t))$ of its average throughput. We assume that $U_i(R_i(t))$ is an increasing concave function.

Define $U(R(t)) = \sum_{i\in\mathcal{M}} U_i(R_i(t))$, and consider maximizing $\lim_{t\to\infty} U(R(t))$. The gradient-based scheduling theory finds the policy that maximizes $U(R(t+1)) - U(R(t))$ for each time t. In our setting, the maximization problem to be solved at each time t is written as

$$\begin{aligned} \max_{x,p} \quad & \sum_{i\in\mathcal{M}} w_i \sum_{j\in\mathcal{N}} x_{ij} \log(1+\eta_{ij} p_j) \\ \text{subject to} \quad & \sum_{j\in\mathcal{N}} p_j \le \bar{P} \\ & \sum_{i\in\mathcal{M}} x_{ij} \le 1, \forall j \in \mathcal{N} \\ & p_j \ge 0, x_{ij} \in \{0,1\}, \forall i \in \mathcal{M}, \forall j \in \mathcal{N} \end{aligned} \quad (2)$$

where $w_i = \left.\frac{dU_i(R_i)}{dR_i}\right|_{R_i=R_i(t)}$, $x = [x_{ij}, \forall i \in \mathcal{M}, \forall j \in \mathcal{N}]$ and $p = [p_j, \forall j \in \mathcal{N}]$. We have dropped t's and B/N, and used $\log$ instead of $\log_2$ without loss of optimality. By using the marginal utility as weight w_i, the optimal solution to the above problem will result in the largest increase of the total average utility, eventually achieving NUM according to the gradient-based scheduling theory. The first and second constraints are total power constraint and subcarrier allocation constraint, respectively. Note that the above problem contains sum rate maximization problem [3] as a special case, i.e., $w_i = 1, \forall i$.

Remark 1. The derivation of the problem (2) is as follows. Take the first order Taylor expansion of $U(R(t+1)) - U(R(t))$ in the neighborhood of $\epsilon_t = 0$, where $\epsilon_t = 1/(t+1)$. The resource allocation problem maximizing the first order term and ignoring high order terms can be written as (2). See [6], [8] for the details. The optimal solution of (2) is obviously a subcarrier and power allocation having maximal utility increment at each time t and roughly speaking, this property results in maximizing the asymptotic utility. The gradient-based scheduling theory proves this mathematically. In brief, once we can find an optimal solution to the problem (2) at each time t, then the average throughput vector $R(t)$ will stochastically converge to an asymptotic optimal point [6]–[8]. Therefore, we need to solve the problem (2) at each time t in order to achieve asymptotic network utility maximization.

Let r_i^t be the rate allocated to user i at time t, i.e., $r_i^t = \sum_{j\in\mathcal{N}} r_{ij}^t$. Denote by F_t the feasible region of $r^t = [r_i^t, \forall i]$ at time t. Let us further define the long-term throughput vector R as $R = \lim_{t\to\infty} R(t)$ if exists. The feasible region of R is defined as the set of long-term throughput vectors, for each point in which there exists a resource allocation policy achieving the point. Roughly speaking, it is a convex combination of all the possible F_t's [7]. Let F denote this region. The snap-shot approach finds a resource allocation maximizing $\sum_i U_i(r_i^t)$ subject to $r^t \in F_t$ at each time t. Therefore, the decision at some time t does not affect those at any other time. The objective of long-term approach is to maximize $\sum_i U_i(R_i)$ subject to $R \in F$. In contrast to snap-shot approach, the decisions at different times are coupled. i.e., the decision at time t affects the one at $t+1$, e.g., through w_i in (2).

Remark 2. Because we are assuming single-cell environment, the optimal resource allocation can be made regardless of other cells' decisions. So, the above problem is no longer valid in multi-cell environment where the decision by one cell can affect the decisions in other cells. This multi-cell issue is beyond the scope of this paper, and we leave it as a future study.

III. Comments on Optimal Algorithm

In this section, we discuss the approaches to find an optimal solution to the problem (2) and how to reduce the search space. The problem is a mixed integer nonlinear programming (MINLP) and unfortunately local optima may not be global optima. If either subcarrier selection or power allocation is fixed, the problem is easy, but it is difficult to solve the problem simultaneously considering both of them. A brute-force approach is the exhaustive search of all possible combinations of subcarrier allocations. The problem formed by each combination is easy because it is a convex optimization with strictly concavity, but the number of combinations will be $O(M^N)$ which is prohibitive in practice. Another approach is to exploit the following lemma showing the equivalence of the relaxed problem (3) to the original problem (2).

$$\begin{aligned} \max_{x,p\ge 0} \quad & \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} w_i x_{ij} \log(1+\eta_{ij} p_j) \\ \text{subject to} \quad & \sum_{j\in\mathcal{N}} p_j \le \bar{P} \\ & \sum_{i\in\mathcal{M}} x_{ij} \le 1, \forall j \in \mathcal{N}. \end{aligned} \quad (3)$$

Lemma 3.1: The solution to the relaxed problem (3) always exists at an extreme point[2] of the feasible set of x in (3).

Proof: First, let x be relaxed to real number variables, i.e., $0 \le x_{ij} \le 1, \forall i, j$ instead of $x_{ij} \in \{0,1\}, \forall i, j$. Since $x_{ij} \le 1$ is a necessary condition for the subcarrier allocation constraint $\sum_{i\in\mathcal{M}} x_{ij} \le 1$, we only need the nonnegativity constraint on x. Thus, the relaxed problem can be written as (3). Let (x^*, p^*) be an optimal solution to (3). Suppose that for some $j \in \mathcal{N}$, there is $\mathcal{M}_j \subseteq \mathcal{M}$ such that $x_{ij}^* > 0, \forall i \in \mathcal{M}_j$. Without loss of generality, assume for some $i \in \mathcal{M}_j$, $w_i \log\left(1+\eta_{ij} p_j^*\right) \ge w_k \log\left(1+\eta_{kj} p_j^*\right), \forall k \in \mathcal{M}_j \setminus \{i\}$. Then, the objective function value with $x_{ij} = 1$ and $x_{kj} = 0, \forall k \in \mathcal{M}_j \setminus \{i\}$ is

[2] An extreme point of a polyhedron is a point (in the polyhedron) which cannot be expressed as a convex combination of two distinct points in the polyhedron. In (3), extreme points are all feasible integral points.

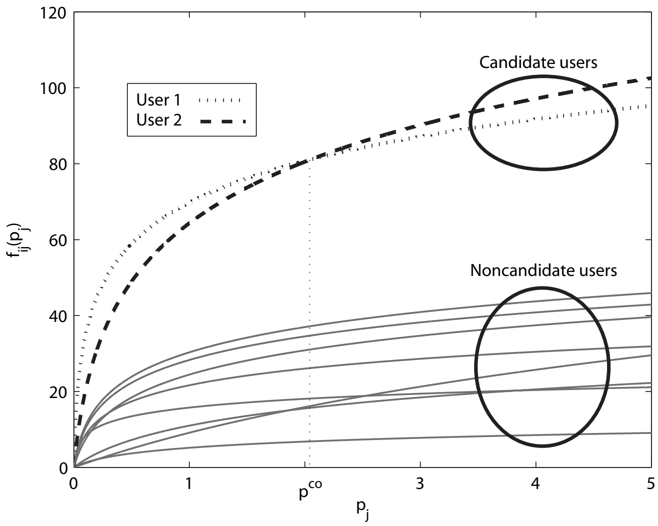


Fig. 1. Example of $f_{ij}(p_j)$'s with $\bar{P} = 5$.

greater than or equal to that with x^*. Therefore, the optimal solution to (3) always exists at an extreme point of the feasible set of x. ■

Intuitively, the solution set of (3) with respect to x is a minimal convex set containing that of (2), implying that it is actually a convex hull of the discrete solution set of (2) which is discrete. Therefore, the extreme points of the solution set of (3) are all feasible to (2), and thus it is possible to find an integral optimal solution by solving the continuous problem (3). Note however that the problem (3) may also have non-integral solution for x, i.e., the problems (3) and (2) may not have the identical solution set. But even in this case, the optimal solutions of (2) are still included in those of (3). Moreover, we can easily get an integral optimal x by mapping the solution vector x of (3), as in the above proof. The problem (3) is a concave maximization for fixed x or for fixed p. This type of problem is called *biconvex programming*, which can be solved by global optimization algorithm (GOP) [17]. However, the algorithm also performs exponentially many number of computations. More details on GOP are discussed in Section IV.

Let us look into the problem further. Define $f_{ij}(p_j) = w_i \log(1 + \eta_{ij} p_j), \forall i \in \mathcal{M}, j \in \mathcal{N}$. Notice that the objective function in (2) is $\sum_{j\in\mathcal{N}} \sum_{i\in\mathcal{M}} x_{ij} f_{ij}(p_j)$. Fig. 1 shows an example of $f_{ij}(p_j)$ for a subcarrier j. We can see that only users 1 and 2 are the candidate users and the other users will not be selected for subcarrier j (Of course, the candidate and noncandidate users can change slot by slot as w_i and η_{ij} can change over time). Therefore we can significantly reduce the search space by eliminating the noncandidate users for each subcarrier, although the problem still possesses exponential complexity.

IV. Suboptimal Algorithm

As discussed above, finding an optimal solution seems to be prohibitive in practice due to its complexity. This section presents a simple suboptimal subcarrier and power allocation algorithm.

A. Proposed Suboptimal Algorithm

We use generalized Benders decomposition for biconvex programming [18]. Applying the decomposition, we can show the following lemma. See Appendix A for the proof.

Lemma 4.1: The following problem is equivalent to the problem (3).

$$\begin{aligned} \max_{x\geq 0,\mu} \quad & \mu \\ \text{subject to} \quad & \mu \leq \max_{p\geq 0} L(x,p,\lambda), \forall \lambda \geq 0 \\ & \sum_{i\in\mathcal{M}} x_{ij} \leq 1, \forall j \in \mathcal{N} \end{aligned} \tag{4}$$

where $L(x,p,\lambda) = \sum_{i\in\mathcal{M}} \sum_{j\in\mathcal{N}} w_i x_{ij} \log(1 + \eta_{ij} p_j) + \lambda \left(\bar{P} - \sum_{j\in N} p_j\right)$.

Denote by $(3)^k$ the problem (3) with some fixed feasible x, say x^k. Due to the restriction on x, solving $(3)^k$ is equivalent to solving (3) with additional constraints. The optimal objective function value of $(3)^k$ thus provides a lower bound to (3). Analogously, let $(4)^k$ denote the problem (4) with fixed feasible λ (say λ^k). In this case, the optimal objective function value of $(4)^k$ provides an upper bound to (4) because the constraints in (4) has been relaxed in $(4)^k$. This property is exploited by global optimization algorithm (GOP) in iterative fashion [17]. In brief, given any iterative algorithm, we can say that the optimal solution has been found when the lower bound by $(3)^k$ becomes equal to the upper bound by $(4)^k$. This approach can be shown to converge in a finite number of iterations [18]. However, each iteration requires a complicated and heavy computation because the inner maximization problem in (4) is defined only implicitly because of x.

We slightly modify (4) to cope with the difficulty, which leads to suboptimality. Notice that the problem (4) becomes infeasible when $\sum_{j\in\mathcal{N}} p_j > \bar{P}$ because the optimal solution μ^* goes to $-\infty$ as λ increases. On the other hand, if p is feasible, i.e., $\sum_{j\in\mathcal{N}} p_j \leq \bar{P}$, it is obvious that the optimal solution is achieved at a point where the second term in $L(x,p,\lambda)$ vanishes. We first solve the outer maximization problem with some initial feasible condition p^0, which converts the problem into scheduling problem for each subcarrier. Let $(4)^0$ denote the problem and m^0 be the subcarrier allocation vector obtained by $(4)^0$, i.e., $m^0(j) = i$ means that subcarrier j is allocated to user i.

Next, solve the inner maximization problem with given subcarrier allocation m^0. Since the inner problem is nothing but (3) with fixed x^0 corresponding to m^0, we denote by $(3)^1$ the inner maximization problem with given m^0. Define p^1 be the optimal power vector to the problem $(3)^1$. Let f_m^0 and f_p^1 be the objective function values of $(4)^0$ and $(3)^1$, respectively. They are then given by $f_m^0 = \sum_{j\in\mathcal{N}} w_{m^0(j)} \log\left(1 + \eta_{m^0(j)j} p_j^0\right)$ and $f_p^1 = \sum_{j\in\mathcal{N}} w_{m^0(j)} \log\left(1 + \eta_{m^0(j)j} p_j^1\right)$. It is obvious that $f_p^1 \geq f_m^0$ because f_m^0 is the objective function value with m^0 and arbitrary p^0 while as f_p^1 is that with m^0 and optimal p^1. Suppose that p^1 is used as initial condition in (4), and respectively denote the problem and its optimal value by $(4)^1$ and f_m^1. Similar to the previous case, we will have $f_m^1 \geq f_p^1$. Doing this iteratively (until no increase in optimal value is achieved)

Algorithm 1 Suboptimal Subcarrier and Power Allocation

1: Initialization: $k = 0$ and $p^k = \bar{P}/N$.
2: While subcarrier allocation is changing,
  2.1 $m^k(j) = \arg\max_{i \in M} w_i \log\left(1 + \eta_{ij} p_j^k\right), \forall j \in N.$
  2.2 solve the following problem

$$\begin{aligned} p^{k+1} = \arg\max_{p \geq 0} \quad & \sum_{j \in N} w_{m^k(j)} \log\left(1 + \eta_{m^k(j)j} p_j\right) \\ \text{subject to} \quad & \sum_{j \in N} p_j \leq \bar{P} \end{aligned} \tag{5}$$

  2.3 $k = k + 1$.
3: (m^{k-1}, p^k) is a suboptimal subcarrier/power allocation.

will yield a local optimal solution. We propose a suboptimal algorithm using $p^0 = \bar{P}/N$ as shown in Algorithm 1. When only one iteration is executed in step 2, the algorithm becomes the suboptimal algorithm proposed in [15]. Unfortunately, we cannot predict how many iterations are needed for the algorithm to stop, but we could see through simulations that it stops within 3 or 4 iterations in most of cases.

B. Simple and Fast Power Allocation

The power allocation problem (5) is usually solved via iterations of which the number is unpredictable. Those iterations could be of too much burden in practice, thus it is desirable to devise an efficient method replacing them. First, it is intuitively clear that the optimality is achieved at

$$\sum_{j \in \mathcal{N}} p_j = \bar{P}. \tag{6}$$

By KKT (Karush-Kuhn-Tucker) condition, we also have $\frac{w_{m(j)}\eta_{m(j)j}}{1+\eta_{m(j)j}p_j} - \lambda = 0$, which actually gives the following $N-1$ equations for some $j \in \mathcal{N}$

$$\frac{w_{m(j)}\eta_{m(j)j}}{1 + \eta_{m(j)j}p_j} = \frac{w_{m(k)}\eta_{m(k)k}}{1 + \eta_{m(k)k}p_k}, \forall k \neq j. \tag{7}$$

(6) and (7) are N linear equations of N variables. Its solution is therefore easily obtained as

$$p_j = w_{m(j)} \frac{\bar{P} + \sum_{k \neq j} \frac{1}{\eta_{m(k)k}} \left\{1 - \frac{w_{m(k)}\eta_{m(k)k}}{w_{m(j)}\eta_{m(j)j}}\right\}}{\sum_{k \in \mathcal{N}} w_{m(k)}} \tag{8}$$

for $j \in \mathcal{N}$. However, the solution (8) could be negative for some j because the nonnegativity constraint on p has not been reflected yet. This can be resolved by eliminating all the subcarriers with non-positive power and solving the linear equations again. In fact, we have to perform this until all the powers become positive.

The natural question one may ask will be "Could any subcarrier (which was non-positive before) achieve positive power after some non-positive subcarriers are eliminated?". Intuitively, this does not happen because the elimination of non-positive subcarrier results in the decrease of effective total power to other subcarriers. Suppose $p_j \leq 0$ and rewrite (6) as $\sum_{k \neq j} p_k = \bar{P} - p_j$. Before the elimination of p_j, the effective total power seen by the subcarriers $k \neq j$ is greater than or equal to $\bar{P}$. But, after the elimination, the effective total power will decrease or remain at the same level. Thus the powers of the remaining subcarriers will be rather non-increasing. This is rigorously proved in Theorem 4.1, of which the proof is given in Appendix B.

Theorem 4.1: Consider subcarrier j, l such that $p_j, p_l \leq 0$, and suppose that only subcarrier l has been eliminated. In this case, p_j is still non-positive.

This theorem shows that within an iteration of the elimination process, we can drop all the subcarriers with non-positive powers at the same time. Therefore, at most N iterations are needed to exactly solve (5).

C. Observations on Equal Power Allocation

We briefly analyze equal power allocation (EPA) based on the results in the above. Let m be an optimal subcarrier allocation. In high SNR regime ($\eta_{m(j)j} \gg 1, \forall j$), the optimality condition (7) can be approximately written as

$$\frac{w_{m(j)}}{p_j} = \frac{w_{m(k)}}{p_k}, \forall k \neq j. \tag{9}$$

Similar to (8), optimal power allocation is obtained as

$$p_j = \frac{\bar{P}}{\sum_{k \in \mathcal{N}} \frac{w_{m(k)}}{w_{m(j)}}}, \forall j \in \mathcal{N}. \tag{10}$$

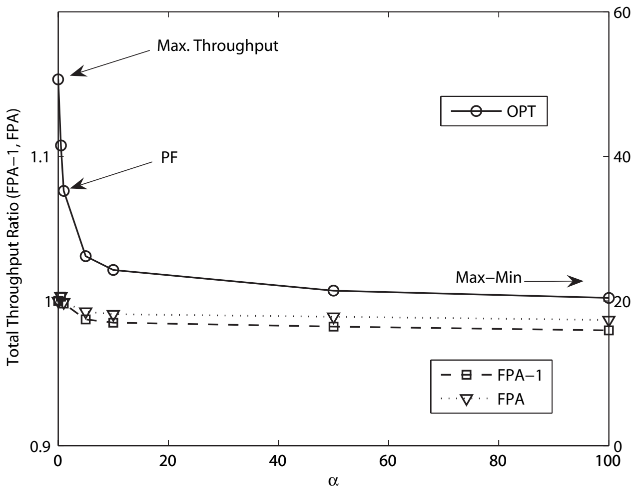
When the objective is to maximize the sum throughput, the utility function is $U_i(R_i(t)) = R_i(t), \forall i \in \mathcal{M}$ so that $w_{m(j)} = 1, \forall j$. It is clear from (10) that the EPA, i.e., $p_j = \bar{P}/N, \forall j$, is approximately optimal. Obviously, the user with the best channel condition is selected over each subcarrier for sum throughput maximization problem. Consequently, the high SNR condition is highly likely to happen in multi-user multi-carrier setting due to statistical effects. The performance gain by optimal frequency-selective power allocation (FPA) could be marginal in this case. We expect that this argument also holds true for efficiency-oriented resource sharing policies under which good channel users are usually scheduled and the dynamic range of $w_{m(j)}$ is not that large.

Suppose that we are using fairness-oriented resource sharing policy. Then, the approximation (9) no longer holds because some bad channel users should be scheduled. Moreover, the dynamic range of $w_{m(j)}$ can become large. The performance gap between FPA and EPA will increase as the difference in weights increases. This argument is validated in Section V. Since the weights change over time, any other static power allocation could also degrade the network performance significantly.

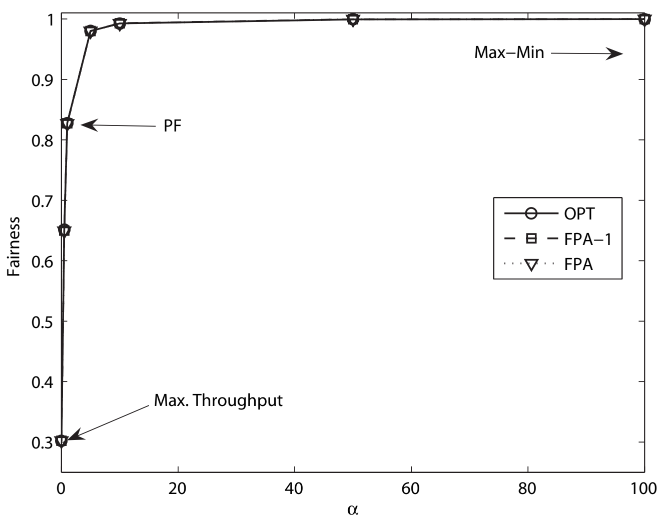
In frequency-flat channels, the EPA is exact optimal irrespective of the types of utility functions. If the channels are frequency-flat, the SNR's of each user over all subcarriers are equal. As a consequence, one user will occupy all the subcarriers. Putting this into (8) yields that allocating equal power to each subcarrier is exact optimal. This is also verified in the simulation. Note that the suboptimal algorithm in Algorithm 1 is also optimal in this case. In brief, the EPA is always optimal in frequency-flat channels or near optimal to the sum capacity maximization problem in high SNR regime. However, the optimality or even approximate optimality of EPA is not guaranteed in other cases.

TABLE I
SUMMARY OF MULTIPATH DELAY-SPREAD MODEL: T_d (DELAY SPREAD), L (NUMBER OF DELAY TAPS)

s	1	2	3	4	5	6	7	8
$T_d(\mu s)$	0	0.5	0.7	2.1	3.7	10	18	20
L	1	10	6	20	6	12	20	6
Name	FF	RA	IO-B	TU	P-B	BU	HT	V-B



(a) T_{F1}/T_O, T_F/T_O, and T_O



(b) Fairness index

Fig. 2. Comparison of optimal and suboptimal algorithms: $s = 5$.

V. Simulation Results

In this section, we present simulation results verifying our arguments and analysis.

A. Simulation Setup

Throughout the simulation, the following utility function [19] is used for each user i

$$U_i(R_i(t)) = \begin{cases} \frac{1}{1-\alpha} R_i(t)^{1-\alpha}, & \text{if } \alpha \geq 0, \alpha \neq 1 \\ \log(R_i(t)), & \text{if } \alpha = 1. \end{cases} \quad (11)$$

This utility function enables to achieve any compromise between efficiency and fairness. When $\alpha = 0$, the total throughput is maximized, and when $\alpha = 1$, the PF (proportional fair) throughput allocation is achieved. As α increases, fairness is improved at the cost of reduced total throughput, and especially as α tends to infinity, the throughput allocation becomes max-min fair. The channels are generated by reflecting path loss, shadowing and fast fading. The path loss (PL) model is $PL(d) = 16.62 + 37.6 \log_{10}(d)$[dB] where d is the distance between a user and the base station in meters. The distances are in $[100, 1000]$m, and the users are assumed to be uniformly distributed around the base station. The shadowing is generated by normal distribution with zero mean and standard deviation $\sigma = 8$dB. For frequency-selective fast fading, we employ standard delay-spread models [20] and define variable s to indicate each model. $s = 1$ especially represents frequency-flat fading, and the summary of the models are given in Table I[3]. Note that the delay spread roughly reflects the degree of frequency-selectivity, but it is not the only factor that determines the selectivity. For each combination of (α, s), 5×10^4 time slots are simulated. The length of a time slot and total power $\bar{P}$ are respectively set to 5ms and 10W.

B. Approximation Quality of Suboptimal Algorithm

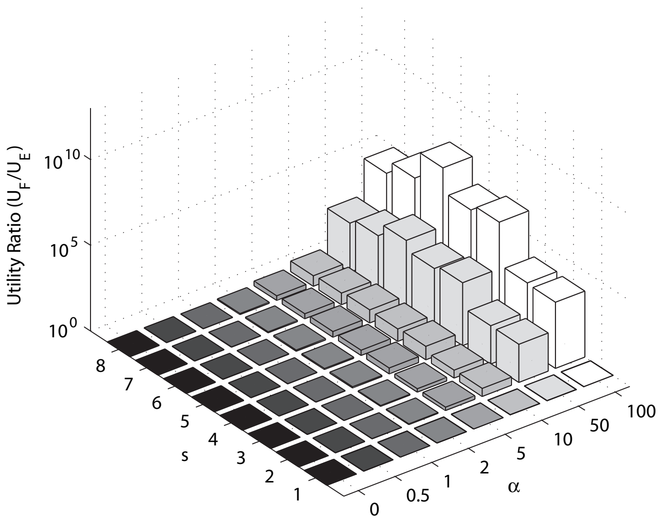
We first examine the performance gap between optimal and suboptimal algorithms. Since finding an optimal solution requires exponentially many number of computations at each time slot, a lightweight scenario with $M = 6$ and $N = 8$ is used. B is set to 20MHz. The results of optimal and suboptimal algorithms will be respectively marked as OPT and FPA. As a special case of FPA, we also show the performance of FPA-1 where only one iteration is carried out in step 2 of Algorithm 1, which is a suboptimal algorithm proposed in [15]. Fig. 2 presents the total throughput (T_O) of optimal algorithm, the total throughput (T_{F1}, T_F) of suboptimal algorithms normalized by T_O, and fairness (Jain's fairness index [21]). We can see that as α increases, T_O decreases while as fairness increases. Our suboptimal algorithm approximates the optimal algorithm within $1 - 2\%$ throughput gap. It can also be observed that FPA-1 and FPA yield almost the same performance. Table II shows that the suboptimal algorithms are near optimal for all cases.

Remark 3. The main interest of this work is not developing and verifying the algorithm, but investigating the suitability of EPA by comparing its performance with that of FPA. It would be ideal if we can use the optimal algorithm for the comparison, but its complexity is so high that for general cases, it is impossible to execute the optimal algorithm in reasonably fast time. For this reason, we need to consider using a simpler suboptimal algorithm and compare its performance loss over optimal algorithm. Of course, the scenario in this simulation is too simple to say that it represents the performance of the suboptimal algorithms in general case. Nevertheless, the result in Fig. 2 and Table II shows the possibility that the suboptimal algorithms FPA-1 and FPA can yield fairly close performance to optimal algorithm. Note also that FPA needs more iterations than FPA-1, but its performance gain over FPA-1 is marginal. Hence, we could use FPA-1 instead of FPA

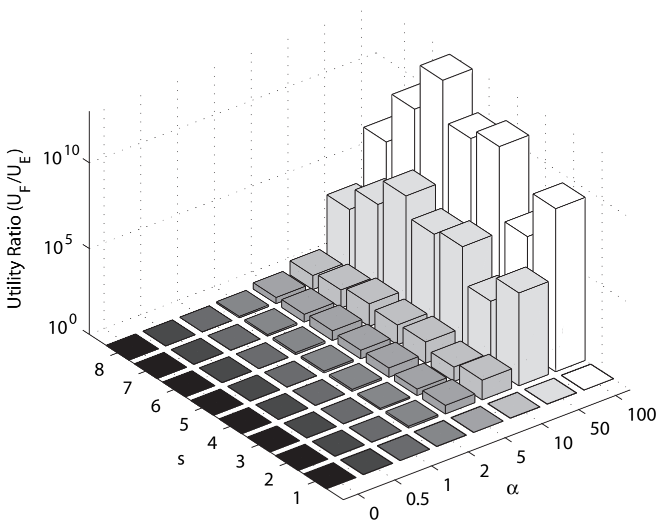
[3]FF(frequency-flat), RA(rural area), IO-B(indoor office B), TU(typical urban), P-B(pedestrian B), BU(bad urban), HT(hilly terrain), V-B(vehicular B)

TABLE II
COMPARISON OF UTILITIES ACHIEVED BY OPTIMAL(OPT) AND SUBOPTIMAL (FPA-1,FPA) ALGORITHMS: $s = 5$ ($x\,e\,y$ MEANS $x \times 10^y$)

α	0.5	1	5	10	50	100
OPT	928.82	51.40	-5.06e-15	-3.18e-33	-2.17e-175	-3.58e-351
FPA-1	928.39	51.38	-5.34e-15	-3.62e-33	-4.70e-175	-2.32e-350
FPA	928.75	51.39	-5.23e-15	-3.44e-33	-3.39e-175	-1.19e-350



(a) Utility ratio (U_F/U_E): $B = 5$MHz



(b) Utility ratio (U_F/U_E): $B = 20$MHz

Fig. 3. Comparison of EPA and FPA for various α and s.

considering complexity, but we will use FPA in order to see the maximum possible performance gap between frequency-selective and equal power allocations.

C. EPA vs. FPA

We test the performance of EPA[4] by comparing with FPA. M and N are fixed to 50 and 512 respectively. The comparison will be shown in three aspects: a) *impact of frequency selectivity*, b) *impact of user SNR distribution*, and c) *impact of discrete rates*. For a), the performances of EPA and FPA

[4]The EPA policy is the execution of step 1 and step 2.1 in Algorithm 1.

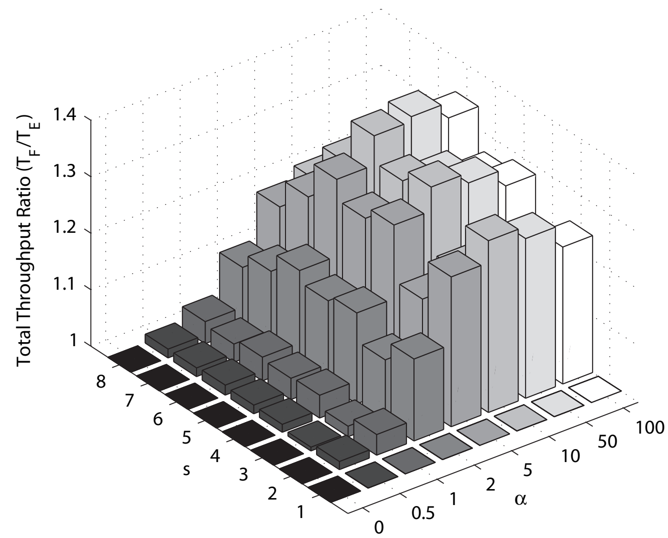


Fig. 4. Total throughput ratio (T_F/T_E): $B = 20$MHz.

are compared for $B = 5$MHz and $B = 20$MHz. It is clear that higher degree of frequency-selectivity is observed when $B = 20$MHz than when $B = 5$MHz. For example, in the environment $s = 2$, the coherence bandwidth is approximately 1MHz(=$\frac{1}{2T_d}$) where T_d is the delay spread. Roughly speaking, the system with $B = 5$MHz experiences 5 variations in frequency domain, and the one with $B = 20$MHz does 20 variations. Thus the latter system has higher selectivity in frequency domain. For b), we first define that a user is in high (resp., low) SNR region when its distance from base station is less (resp., greater) than 600m. The average received SNR per unit power (averaged over time and frequency) is 5dB at d =600m. The simulation is conducted by varying the percentage of low SNR users. For c), the rates are quantized with various quantization levels. Since only the discrete rates are available in practice, it is worthy of observing the impact of discrete rates on the performance of EPA.

1) Impact of Frequency Selectivity: Fig. 3(a) plots the utility ratio U_F/U_E with $B = 5$MHz for all combinations of (α, s) where U_F and U_E are the utilities achieved by FPA and EPA, respectively. Because the utility becomes negative for $\alpha > 1$, we compute the ratio by dividing U_E by U_F in that case. But for simplicity, we denote the ratio by U_F/U_E in all cases. Observe that the performance gap between FPA and EPA is negligible for small α, i.e., efficiency-oriented policies. Since our algorithm finds an exact optimal solution for $\alpha = 0$, the EPA policy is near optimal to the total throughput maximization problem, which is expected in Section IV-C. Observe also that as α increases the performance gain by FPA grows. When α is small, the difference in marginal utilities, i.e., w_i's is relatively small. Hence, the subcarrier allocation mainly depends on SNR so that the user with high SNR will be selected for each subcarrier. As a result, the SNR $\eta_{m(j)j}$ per unit power for each $j \in \mathcal{N}$ will be high. As we discussed in Section IV-C, the EPA approximates optimal frequency-selective power allocation fairly well. But, as α increases or equivalently fairness is emphasized, the dynamic range of w_i becomes very large so that the user with bad channel condition can be scheduled. The EPA is no longer close to optimal in this case. This is why we observe the performance trend shown in Fig. 3(a). A similar tendency is observed with respect to s. See especially that the EPA is exact optimal for frequency-flat channels ($s = 1$) irrespective of the value of α. In contrast, the performance improvement by our FPA over EPA is substantial for frequency-selective channels ($s \neq 1$).

Fig. 3(b) depicts the results with $B = 20$MHz. The gain by FPA has significantly increased compared to the previous case, which implies that the performance of EPA deteriorates more in broader bandwidth (or more frequency-selective environments). Considering that the future wireless system is likely to adopt broad bandwidth, the significance of frequency-selective power allocation over static equal power allocation will be more emphasized in the future. Fig. 4 shows the total throughput ratio T_F/T_E where T_E is the total throughput achieved by EPA. The throughput ratio also follows the characteristics in the utility ratio. This result implies that under fairness-oriented policies, FPA has significant gain over EPA in not only optimality but also efficiency perspective. All other utility ratios that will be shown have similar throughput ratios, but we will not show the throughput ratio for some results due to limited space.

2) Impact of SNR Distribution: Fig. 5 shows the utility ratio for different SNR distributions and α's. When all the users are in high SNR region, the gain by FPA over EPA is marginal for all α's. As discussed above, the EPA achieves comparable performance to our FPA for $\alpha = 0$. When $\alpha \neq 0$, it is still true that both η_{ij} and w_i affect the subcarrier allocation, but the user selected for each subcarrier will probably have high SNR at that subcarrier because all the users are in high SNR region. The approximated power allocation (10) is valid in this case. If α is small, the difference of w_i among users will be relatively small so that (10) produces almost equal power allocation. If α is large, the throughput allocation approaches max-min fairness, i.e., equal throughput allocation. This in turn implies equal w_i's. Consequently, (10) produces almost equal power allocation. This is why the performance gain by our FPA over EPA is marginal in high SNR regime. The result for the case of mixed high SNR and low SNR users can be interpreted as in Section V-C1.

Another interesting result is the performance gain when all the users are in low SNR region. For throughput maximization (,i.e., $\alpha = 0$), the EPA is not as near optimal as in all the previous cases (about 10% gain by FPA), although it is hard to see in the figure because the ratio goes up to 10^{20} in some cases. This reasserts that the EPA is near optimal to throughput maximization problem in high SNR regime, but not in low SNR regime. It should be noted that if the high SNR criterion (currently, 5dB) is set to a lower value, the performance gain by FPA would increase in low SNR regime. For other values of α, we have similar results to the previous cases.

3) Impact of Discrete Rates: So far we have assumed that the continuous rate is achievable, but only finite number of discrete rates are available in practice. It will be thus interesting to see how this affects the performance of EPA. For this, the following quantization is used in the achievable data rate:

$$q \cdot \left\lfloor \frac{B}{N} \log_2(1 + \eta_{ij} p_j)/q \right\rfloor \qquad (12)$$

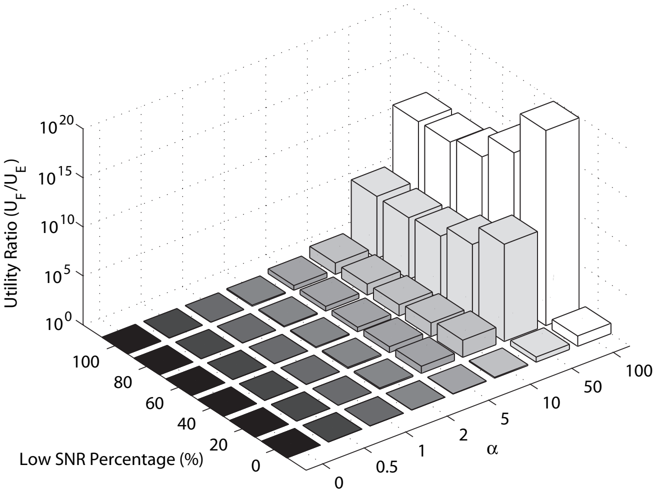


Fig. 5. Comparison of EPA and FPA for various α and SNR distributions: $s = 6$ and $B = 20$MHz.

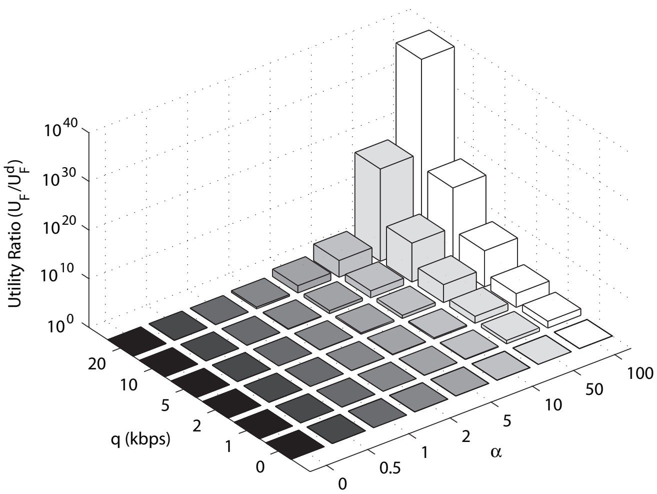


Fig. 6. Performance degradation of FPA (U_F/U_F^d): continuous vs. discrete ($s = 6$ and $B = 20$MHz).

where q is a quantization level. We apply this quantization for subcarrier and power allocation found by Algorithm 1 and EPA (which assume continuous achievable data rate). This is different from existing modulations, e.g., M-QAM (M-ary quadrature amplitude modulation) where the set of available rates is fixed beforehand. Nevertheless, (12) will sufficiently reflect the impact of such discrete rates. The dynamic range of the achievable data rate over a subcarrier is about (0,400)kbps. So, for example, $q = 100$kbps indicates that there are 4 available rates including 0, 100, 200 and 300kbps.

Fig. 6 plots the utility ratio U_F/U_F^d of FPA where U_F and U_F^d are the utility of FPA with continuous rate and discrete rate, respectively. The value of q is varied from 0 to 20kbps where $q = 0$ means continuous achievable data rate. We can see that as the quantization level increases the performance of FPA is degraded compared to the case of continuous rate.

We first show the throughput ratio T_F^d/T_E^d in Fig. 7 where T_E^d is the total throughput of EPA with discrete rates. The performance gap between FPA and EPA is minimal for continuous rate ($q = 0$), and as the quantization level increases, the performance of EPA is more degraded. Observe especially

TABLE III
COMPARISON OF UTILITIES ACHIEVED BY EPA AND FPA UNDER DISCRETE RATES: $s = 6$ AND $B = 20$MHz ($x\,e\,y$ MEANS $x \times 10^y$)

	α	0.5	1	5	10	50	100
q=1	EPA	2.1e3	278.3	-4.1e-8	-1.1e-18	-7.6e-101	-1.5e-202
	FPA	2.2e3	284.8	-1.4e-8	-7.4e-20	-1.5e-107	-4.7e-216
q=5	EPA	2.0e3	224.1	-3.0e-2	-3.3e-4	-6.3e-17	-5.8e-32
	FPA	2.1e3	281.3	-2.3e-8	-2.5e-19	-1.4e-104	-5.1e-210
q=20	EPA	497.4	-17.9	-5.7e6	-2.4e14	-5.2e77	-3.3e157
	FPA	2.0e3	261.1	-3.8e-7	-1.5e-16	-2.9e-89	-3.6e-179

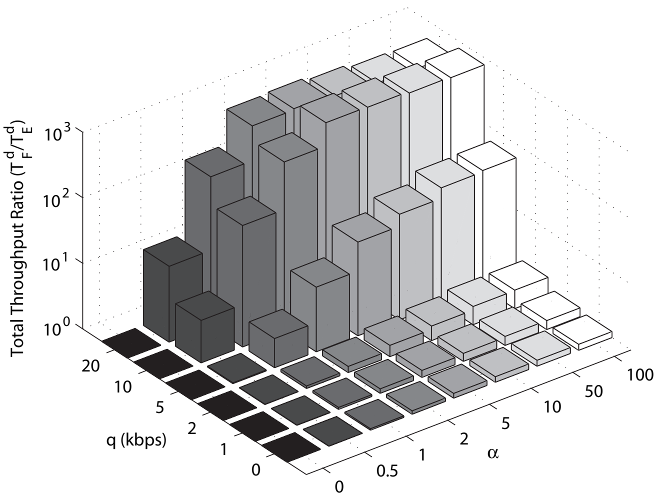


Fig. 7. Impact of discrete rates on EPA (T_F^d/T_E^d): FPA vs. EPA ($s = 6$ and $B = 20$MHz).

that for $q \geq 5$kbps the throughput of EPA is degraded to an unacceptable level because it goes down by factor of tens to hundreds compared to that of FPA. In other words, the EPA is completely malfunctioning for high quantization level. Table III shows the utilities achieved by EPA and FPA. We can see the similar results to the total throughput ratio. Especially, the gain is enormous for the case of PF ($\alpha = 1$) which is the most commonly used scheduler. This shows the necessity of frequency-selective power allocation not only in theory but also in practice.

Although we used particular form of utility function (11), the results with other increasing concave utility functions will be similar to those shown here. In brief, we can conclude that the equal power allocation could be an inappropriate choice and frequency-selective power allocation is needed in general utility maximization problem.

VI. CONCLUSIONS

In this paper, we revisited the equal power allocation which has been commonly used to relax the difficulty of resource allocation problem in multi-carrier systems. To that end, we tested its performance through the comparison with frequency-selective power allocation in asymptotic NUM problem. We concluded that the equal power allocation shows excellent performance under efficiency-oriented resource sharing policies, but its performance is severely degraded under fairness-oriented policies. Thus, if fairness and performance are of concern, we recommend that one would use frequency-selective power allocation even at the cost of increased computational burden. In order to verify our arguments, we presented extensive simulation results in term of resource sharing policy, frequency selectivity, user SNR distribution and discrete rates.

APPENDIX A
PROOF OF LEMMA 4.1

It follows from projection onto x [18] that the relaxed problem (3) is equivalent to

$$\begin{aligned} \max_{x\geq 0} \quad & v(x) \\ \text{subject to} \quad & v(x) = \max_{p\geq 0} \quad \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} w_i x_{ij}\log\left(1+\eta_{ij}p_j\right) \\ & \qquad \text{subject to} \quad \sum_{j\in\mathcal{N}} p_j \leq \bar{P} \\ & \sum_{i\in\mathcal{M}} x_{ij} \leq 1, \forall j\in\mathcal{N}. \end{aligned} \tag{13}$$

By strong duality, the optimal value of the inner maximization problem in (13) is equal to that of the minimization of its dual function. As a result, (13) can be written as

$$\begin{aligned} \max_{x\geq 0} \quad & v(x) \\ \text{subject to} \quad & v(x) = \min_{\lambda\geq 0}\max_{p\geq 0} L(x,p,\lambda) \\ & \sum_{i\in\mathcal{M}} x_{ij} \leq 1, \forall j\in\mathcal{N} \end{aligned} \tag{14}$$

where $L(x,p,\lambda) = \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}} w_i x_{ij}\log\left(1+\eta_{ij}p_j\right) + \lambda\left(\bar{P} - \sum_{j\in\mathcal{N}} p_j\right)$. It is easy to see that the optimal solution to the problem (14) can be obtained by replacing the objection function in (14) with new variable μ and adding the constraint $\mu \leq v(x)$. This new constraint implies that μ should be less than or equal to the minimum value of $\max_{p\geq 0} L(x,p,\lambda)$ over all $\lambda \geq 0$, it can be rewritten as $\mu \leq \max_{p\geq 0} L(x,p,\lambda), \forall \lambda 0$. This shows the equivalence between (14) and (4), thereby completing the proof.

APPENDIX B
PROOF OF THEOREM 4.1

After elimination of l, p_j will be

$$p_j = w_{m(j)} \frac{\bar{P} + \sum_{k\neq j,l} \frac{1}{\eta_{m(k)k}} \left\{1 - \frac{w_{m(k)}\eta_{m(k)k}}{w_{m(j)}\eta_{m(j)j}}\right\}}{\sum_{k\neq l} w_{m(k)}}. \tag{15}$$

The numerator in (15) can be rewritten as

$$\bar{P} + \sum_{k\neq j} \frac{1}{\eta_{m(k)k}} \left\{1 - \frac{w_{m(k)}\eta_{m(k)k}}{w_{m(j)}\eta_{m(j)j}}\right\} - \frac{1}{\eta_{m(l)l}} \left\{1 - \frac{w_{m(l)}\eta_{m(l)l}}{w_{m(j)}\eta_{m(j)j}}\right\}. \tag{16}$$

First, assume $w_{m(j)}\eta_{m(j)j} \geq w_{m(l)}\eta_{m(l)l}$. The first two terms in (16) are non-positive because they are the numerator of p_j before the elimination of l. The last term is also non-positive by the assumption. Hence, p_j is still non-positive in this case.

Assume otherwise, i.e., $w_{m(j)}\eta_{m(j)j} < w_{m(l)}\eta_{m(l)l}$. From the non-positivity of p_j before the elimination of l, we can show

$$w_{m(j)}\eta_{m(j)j} \leq \frac{\sum_{k\neq j} w_{m(k)}}{\bar{P} + \sum_{k\neq j} \frac{1}{\eta_{m(k)k}}}. \tag{17}$$

The numerator in (15) can be rewritten as

$$\begin{aligned} &\bar{P} + \textstyle\sum_{k\neq j,l} \frac{1}{\eta_{m(k)k}} - \frac{1}{w_{m(j)}\eta_{m(j)j}} \textstyle\sum_{k\neq j,l} w_{m(k)} \\ &\le \bar{P} + \textstyle\sum_{k\neq j,l} \frac{1}{\eta_{m(k)k}} - \left(\textstyle\sum_{k\neq j,l} w_{m(k)}\right)\left(\frac{\bar{P}+\sum_{k\neq j}\frac{1}{\eta_{m(k)k}}}{\sum_{k\neq j} w_{m(k)}}\right) \end{aligned} \tag{18}$$

where the inequality follows from (17). Rearranging (18) yields

$$w_{m(l)} \frac{\bar{P} + \sum_{k\neq j,l} \frac{1}{\eta_{m(k)k}} \left\{1 - \frac{w_{m(k)}\eta_{m(k)k}}{w_{m(l)}\eta_{m(l)l}}\right\}}{\sum_{k\neq j} w_{m(k)}}. \tag{19}$$

Similar to the first case, the numerator in (19) can be divided into

$$\bar{P} + \sum_{k\neq l} \frac{1}{\eta_{m(k)k}} \left\{1 - \frac{w_{m(k)}\eta_{m(k)k}}{w_{m(l)}\eta_{m(l)l}}\right\} - \frac{1}{\eta_{m(j)j}} \left\{1 - \frac{w_{m(j)}\eta_{m(j)j}}{w_{m(l)}\eta_{m(l)l}}\right\}. \tag{20}$$

The first two terms are non-positive because it is the numerator of p_l before its elimination, and the last term is negative by the assumption $w_{m(j)}\eta_{m(j)j} < w_{m(l)}\eta_{m(l)l}$. Hence, (15) is also negative in this case. This establishes the theorem.

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