Design of a CLOS Guidance Law Via Feedback Linearization

IN-JOONG HA
SONG CHONG
Seoul National University

This paper describes an application of the recently developed feedback linearization technique to the design of a new command to line-of-sight (CLOS) guidance law for short-range surface-to-air missiles. The key idea lies in converting the three-dimensional CLOS guidance problem to the tracking problem of a time-varying nonlinear system. Our result may shed new light on the role of the feedforward acceleration terms used in the conventional CLOS guidance laws. Through computer simulation, we investigate the effect of the dynamics of the ground tracker and the autopilot on the guidance performance of our new CLOS guidance law.

Manuscript received October 30, 1990, revised February 6, 1991.

IEEE Log No. 9103604.

Authors' address: Dept. of Control and Instrumentation Engineering and Automation and Systems Research Institute, Seoul National University, San 56-1, Shinlim-Dong, Kwanak-Gu, Seoul, 151-742, Korea.

0018-9251/92/$3.00

I. NOMENCLATURE

ψ_t	Yaw angle of target
θ_t	Pitch angle of target
ψ_m	Yaw angle of missile
θ_m	Pitch angle of missile
ϕ_{mc}	Roll angle command
σ_t	Azimuth angle of line-of-sight (LOS) to target
γ_t	Elevation angle of LOS to target
σ_m	Azimuth angle of LOS to missile
γ_m	Elevation angle of LOS to missile
$\Delta\sigma$	$\sigma_m - \sigma_t$
$\Delta\gamma$	$\gamma_m - \gamma_t$
T	Thrust force
D	Drag force
M	Mass of missile
g	Gravity acceleration
a_x	Axial acceleration of missile
a_{yc}	Yaw acceleration command
a_{zc}	Pitch acceleration command
a_{ty}	Yaw acceleration of target
a_{tz}	Pitch acceleration of target
R_m	Missile range from ground tracker
R_t	Target range from ground tracker
v_m	Missile velocity
v_t	Target velocity
a_t	Target acceleration
$\mathbf{v}_t$	Target velocity vector
$\mathbf{a}_t$	Target acceleration vector
$s\theta$	$\sin\theta$
$c\theta$	$\cos\theta$
$(x_1 \ldots x_n)$	Column vector with scalar components x_i, $i = 1,\ldots,n$
$[x_1,\ldots,x_n]$	Row vector with scalar components x_i, $i = 1,\ldots,n$
R^+	Nonnegative real line
$\lvert x\rvert$	Euclidean norm of vector x
$\lVert A\rVert$	Induced norm of matrix A
$\hat{\lVert} A \hat{\rVert}$	Frobenius norm of matrix A
(X_I, Y_I, Z_I)	Inertial frame
(X_M, Y_M, Z_M)	Missile body frame
(X_L, Y_L, Z_L)	LOS frame
(x_t, y_t, z_t)	Target position in inertial frame
(x_m, y_m, z_m)	Missile position in inertial frame
(R_P, e_1, e_2)	Missile position in LOS frame
$\mathbf{i}_M$	Unit vector corresponding to X_M axis
$\mathbf{i}_I, \mathbf{j}_I, \mathbf{k}_I$	Unit vectors corresponding, respectively, to X_I, Y_I, Z_I axes
$\mathbf{i}_L, \mathbf{j}_L, \mathbf{k}_L$	Unit vectors corresponding, respectively, to the X_L, Y_L, Z_L axes

II. INTRODUCTION

The principle of command to line-of-sight (CLOS) guidance [4, 5, 11, 14, 20–22] is to force the missile to fly as nearly as possible along the instantaneous line joining the ground tracker and the target, which is called the line-of-sight (LOS). The CLOS guidance has been regarded as a low-cost guidance concept because it emphasizes placement of avionics on the launch platform, as opposed to on board the expendable weapon. Recent advances in beam-pointing technology have led to renewed interest in CLOS guidance. The performance of CLOS guidance for short-range engagements is known to be typically good. In the case of medium-range or long-range engagements, however, the quality of the ground tracker fundamentally limits the performance of CLOS guidance.

The CLOS guidance laws for skid-to-turn (STT) missiles are composed of two lateral acceleration commands, pitch and yaw. Each of them is shaped by the sum of an error compensation acceleration term to null deviation of the missile from the LOS to target and a feedforward acceleration term to make the missile chase the LOS rotation. The feedforward acceleration term plays an important role when large LOS rates occur as in the case with CLOS guidance for short-range air defense intercept scenarios. In the earlier results [5, 21, 22], however, this feedforward acceleration term was derived approximately under some restrictive assumptions. In [5, 21], it was assumed that the missile is on the LOS to target and its velocity vector lies on the so called flyplane. In [22], the lateral acceleration of the missile projection point onto the LOS to target was adopted as the feedforward acceleration term. A different approach to CLOS guidance can be found in [4, 11, 14], where linear stochastic optimal control theory was used to take into account autopilot dynamics, LOS measurement noise, and/or the limit in aerodynamic control surface.

We focus our efforts on elucidating the precise expression and role of feedforward acceleration term for the case of general pursuit situation. We first show that the general three-dimensional CLOS guidance problem can be converted to a nonlinear tracking problem. This constitutes our key idea. Thereby, the recently developed feedback linearization technique [1, 8, 10, 12, 13, 16] can be easily applied to the CLOS guidance problem. We propose a new CLOS guidance law. It involves a term which is, in the form, similar to the feedforward acceleration term used in the earlier results [5, 21, 22]. This term in our new CLOS guidance law is required to transform the nonlinear tracking problem into a linear one, while the feedforward acceleration term in the earlier results is used to make the missile chase the LOS rotation. Thus, our result may shed new light on the role of the feedforward acceleration terms used in the earlier CLOS guidance laws. It is shown that our CLOS guidance law can drive miss distance to zero against a randomly maneuvering target in the three-dimensional space. It is, however, computationally complex. In this context, we attempt to simplify the new CLOS guidance law so that the computational burden can be reduced without significant performance degradation.

Finally, we note that the feedback linearization technique has been applied recently to control highly nonlinear aerospace systems. For instance, it was applied to aircraft flight control [15, 17, 18, 23], spacecraft attitude control [2, 3, 19, 24], and missile autopilot [25]. To the authors' knowledge, however, this work presents the first result to apply the feedback linearization technique to missile guidance.

III. PROBLEM FORMULATION

In this section, we show that the three-dimensional CLOS guidance problem can be formulated as a tracking problem of a time-varying nonlinear system.

In modeling the pursuit dynamics of missile and target, we assume the following.

A1: Compared with the overall guidance loop, the autopilot and ground tracker dynamics are fast enough to be neglected.
A2: The total angle-of-attack is small enough to be neglected.

These assumptions have been generally accepted in the design and analysis of missile guidance laws. However, as will be discussed soon, the assumption A2 is not the precondition for the desired performance of our new CLOS guidance law but is introduced only for simplicity of our developments. The effect of significant autopilot and ground tracker dynamics on guidance performance is investigated through simulations.

The three-dimensional pursuit situation is depicted in Fig. 1. The origin of the inertial frame is located at the ground tracker. The Z_I axis is vertical upward and the $X_I - Y_I$ plane is tangent to the Earth surface. The origin of the missile body frame is fixed at the missile center of mass with the X_M axis forward along the missile centerline. Under the prescribed assumptions, motion of the missile in the inertial frame can be represented by

$$\begin{aligned}\ddot{x}_m &= a_x c\theta_m c\psi_m - a_{yc}(s\phi_{mc}s\theta_m c\psi_m + c\phi_{mc}s\psi_m) \\ &\quad - a_{zc}(c\phi_{mc}s\theta_m c\psi_m - s\phi_{mc}s\psi_m) \\ \ddot{y}_m &= a_x c\theta_m s\psi_m - a_{yc}(s\phi_{mc}s\theta_m s\psi_m - c\phi_{mc}c\psi_m) \\ &\quad - a_{zc}(c\phi_{mc}s\theta_m s\psi_m + s\phi_{mc}c\psi_m) \\ \ddot{z}_m &= a_x s\theta_m + a_{yc}s\phi_{mc}c\theta_m + a_{zc}c\phi_{mc}c\theta_m - g\end{aligned} \quad (1)$$

$$\begin{aligned}\dot{\psi}_m &= a_{yc}c\phi_{mc}/(v_m c\theta_m) - a_{zc}s\phi_{mc}/(v_m c\theta_m) \\ \dot{\theta}_m &= a_{yc}s\phi_{mc}/v_m + a_{zc}c\phi_{mc}/v_m - gc\theta_m/v_m.\end{aligned} \quad (2)$$

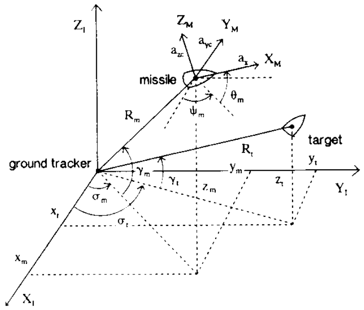

Fig. 1. Three-dimensional pursuit situation.

Here, v_m is the velocity of the missile given by

$$v_m \triangleq (\dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2)^{1/2} \tag{3}$$

and a_x is the axial acceleration of the missile given by

$$a_x \triangleq (T - D)/M. \tag{4}$$

Note that, by the assumption A1, the autopilot commands a_{yc}, a_{zc}, and ϕ_{mc} directly appear in the above dynamic equations of missile motion. Also, note that the assumption A2 is used only in deriving a simplified model (2) of missile attitude dynamics. As is seen in Section V and VI, the derivation of our CLOS guidance laws (39), (42) does not rely on the simplified model (2) of missile attitude dynamics. Hence, the omission of A2 does not result in any degradation of guidance performance. Definitions of the symbols and notations used frequently in our developments are given in the Nomenclature.

Next, we define a tracking error in order to convert the CLOS guidance problem to a tracking problem. As mentioned in Section II, the concept of CLOS guidance is to guide the missile onto the LOS to target. Therefore a reasonable choice of tracking error may be

$$e \triangleq \begin{bmatrix} \Delta\sigma \\ \Delta\gamma \end{bmatrix}. \tag{5}$$

Even in the case of small tracking error, however, this selection can cause large miss distance as the missile flies farther from the launch point.

To overcome this shortcoming, we consider a different choice of tracking error. We first define the LOS frame as is shown in Fig. 2. Its origin is located at the ground tracker. The X_L axis forwards along the LOS to target and the Y_L axis is horizontally directed to the left of the $X_L - Z_L$ plane. Then, the coordinates (R_P, e_1, e_2) indicated in Fig. 2 represent the missile position in the LOS frame. We define the tracking error as

$$e \triangleq \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \triangleq \begin{bmatrix} -x_m s\sigma_t + y_m c\sigma_t \\ -x_m s\gamma_t c\sigma_t - y_m s\gamma_t s\sigma_t + z_m c\gamma_t \end{bmatrix}. \tag{6}$$

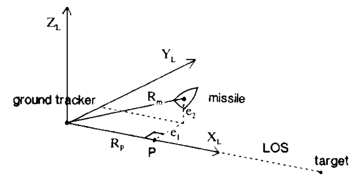

Fig. 2. Definition of tracking error.

Note that $|e|$ just represents the smallest distance from the missile to the LOS to target. Therefore, the missile will eventually hit the target if the tracking error is driven to zero before the target crosses the missile.

Siegel and Lee [22] considered the same form of tracking error as the one in (6). In [4, 11, 14], the so-called pseudotarget was defined to lie on the LOS to target at the same distance from the ground tracker as the missile. Then, the tracking error was chosen as the Y_I axis and Z_I axis components of the line segment joining the pseudotarget and the missile. Such choice of tracking error facilitates the application of linear stochastic optimal control theory to CLOS guidance. In the usual situation with small LOS error angles (here, $\Delta\sigma$ and $\Delta\gamma$), the above two types of tracking error are approximately identical.

So far, we have shown that the three-dimensional CLOS guidance problem can be formulated as a tracking problem. Let

$$x \triangleq (x_1, \ldots, x_8) \triangleq (x_m, y_m, z_m, \dot{x}_m, \dot{y}_m, \dot{z}_m, \psi_m, \theta_m) \in R^8,$$

$$u \triangleq (u_1, u_2) \triangleq (a_{yc}, a_{zc}) \in R^2,$$

and

$$y_d \triangleq (y_{d1}, y_{d2}) \triangleq (\sigma_t, \gamma_t) \in R^2.$$

Using these notations and regarding the axial acceleration of the missile as a time function, we can write (1), (2), and (6) in the following state space form:

$$\dot{x} \triangleq f_0(x,t) + \sum_{i=1}^{2} f_i(x) u_i; \qquad e \triangleq E(x, y_d), \tag{7}$$

where the functions $f_0 : R^8 \times R^+ \to R^8$, $f_i : R^8 \to R^8$, $i = 1,2$; and $E : R^8 \times R^2 \to R^2$ are defined by

$$f_0(x,t) \triangleq (x_4, x_5, x_6, a_x(t) c x_7 c x_8, a_x(t) s x_7 c x_8, a_x(t) s x_8 - g, 0, -g c x_8/(x_4^2 + x_5^2 + x_6^2)^{1/2}) \tag{8}$$

$$f_1(x) \triangleq (0,0,0,-s\phi_{mc}sx_8cx_7 - c\phi_{mc}sx_7,$$
$$-s\phi_{mc}sx_8sx_7 + c\phi_{mc}cx_7,$$
$$s\phi_{mc}cx_8, c\phi_{mc}/\{(x_4^2+x_5^2+x_6^2)^{1/2}cx_8\},$$
$$s\phi_{mc}/(x_4^2+x_5^2+x_6^2)^{1/2}) \quad (9)$$

$$f_2(x) \triangleq (0,0,0,-c\phi_{mc}sx_8cx_7 + s\phi_{mc}sx_7,$$
$$-c\phi_{mc}sx_8sx_7 - s\phi_{mc}cx_7,$$
$$c\phi_{mc}cx_8, -s\phi_{mc}/\{(x_4^2+x_5^2+x_6^2)^{1/2}cx_8\},$$
$$c\phi_{mc}/(x_4^2+x_5^2+x_6^2)^{1/2}) \quad (10)$$

$$E(x,y_d) \triangleq (-x_1s\sigma_t + x_2c\sigma_t, -x_1s\gamma_tc\sigma_t$$
$$-x_2s\gamma_ts\sigma_t + x_3c\gamma_t). \quad (11)$$

Now, the tracking problem we want to solve is to find a control law u to drive the tracking error e to zero.

IV. FEEDBACK LINEARIZING APPROACH TO TRACKING IN NONLINEAR SYSTEMS

In this section, we describe our approach to the tracking problem formulated in the preceding section. Our approach is basically motivated by the recently developed input–output linearization technique [8, 10, 12, 13, 16]. The general idea of the input–output linearization technique is to linearize the input–output dynamic characteristics of a nonlinear system via an appropriate nonlinear feedback control law. This input–output linearization technique facilitates the controller design of nonlinear systems since the resulting systems have linear input–output dynamic characteristics. This technique has been extended to the tracking problem of nonlinear systems [6, 9].

The practical use of the input–output linearization technique has been severely limited by the difficulties in verifying the conditions for input–output linearization and solving a set of partial differential equations to obtain the desired control law and state transformation. However, there is a special class of nonlinear systems, for which it is not necessary to solve a set of partial differential equations [1, 8, 13]. In the similar way, we can characterize the special class of tracking problems, to which the approach proposed in [9] can be easily applied. For readable development of our results, we describe it here although its characterization is easily deducible from the results in [1, 8, 13].

Consider the following nonlinear tracking problem:

$$\dot{x} = f_0(x,t) + \sum_{i=1}^{m} f_i(x,t)u_i; \qquad e = E(x,y_d) \quad (12)$$

where $f_i : R^n \times R^+ \to R^n$, $u_i(t) \in R$, $i = 1,\ldots,m$; $f_0 : R^n \times R^+ \to R^n$, $E : R^n \times R^s \to R^m$, $y_d : R^+ \to R^s$; $x(t) \in R^n$, $e(t) \in R^m$, and $t \in R^+$. Here, the time function y_d represents the desired trajectory of the system output. For the system (12), define the vector fields X_0 and X_i, $i = 1,\ldots,m$ by

$$X_0 \triangleq \partial/\partial t + \sum_{j=1}^{n} f_{0,j}(x,t)\partial/\partial x_j$$
$$X_i \triangleq \sum_{j=1}^{n} f_{i,j}(x,t)\partial/\partial x_j, \qquad i = 1,\ldots,m \quad (13)$$

where $f_{i,j}$, x_j are the jth components of f_i, x, respectively.

Suppose that the nonlinear system (12) satisfies the following conditions.

B1: There exist nonnegative integers d_i, $i = 1,\ldots,m$ such that

$$[X_1X_0^kE_i(x,y_d(t))\ldots X_mX_0^kE_i(x,y_d(t))] = 0,$$
$$x \in R^n, \quad t \in R^+, \quad k = 0,\ldots,d_i - 1 \quad (14)$$
$$[X_1X_0^{d_i}E_i(x,y_d(t))\ldots X_mX_0^{d_i}E_i(x,y_d(t))] \neq 0,$$
$$x \in R^n, \quad t \in R^+ \quad (15)$$

and

B2:

$$y_d \in C^{d_0} \qquad \text{where} \quad d_0 = \max\{d_i + 1,\ i = 1,\ldots,m\}.$$

Note that when

$$[X_1E_i(x,y_d(t))\ldots X_mE_i(x,y_d(t))] \neq 0,$$
$$x \in R^n, \quad t \in R^+ \quad (16)$$

the condition B1 is still satisfied with $d_i = 0$.

Define the functions D^*, A^* by

$$D^*(x,t,Y_d(t)) \triangleq \begin{bmatrix} D_1^*(x,t,Y_d(t)) \\ \vdots \\ D_m^*(x,t,Y_d(t)) \end{bmatrix} \quad (17)$$

where

$$D_i^*(x,t,Y_d(t)) \triangleq [X_1X_0^{d_i}E_i(x,y_d(t))\ldots X_mX_0^{d_i}E_i(x,y_d(t))],$$
$$i = 1,\ldots,m \quad (18)$$

and

$$A^*(x,t,Y_d(t)) \triangleq (X_0^{d_1+1}E_1(x,y_d(t)),\ldots,X_0^{d_m+1}E_m(x,y_d(t))). \quad (19)$$

Here $Y_d(t) \triangleq (y_d(t), y_d^{(1)}(t),\ldots,y_d^{(d_0)}(t)) \in R^{s(d_0+1)}$ and $y_d^{(j)}$ is the jth derivative of y_d.

Now, we show that the control law and state transformation for input–output linearization of the system (12) can be easily found if the following condition is satisfied in addition to B1 and B2.

B3:

$$D^*(x,t,Y_d(t)) \text{ is nonsingular}, \qquad x \in R^n, \quad t \in R^+.$$

Define the mappings α, β, and T by

$$\alpha(x,t,Y_d(t)) \triangleq -[D^*(x,t,Y_d(t))]^{-1}A^*(x,t,Y_d(t)) \tag{20}$$

$$\beta(x,t,Y_d(t)) \triangleq [D^*(x,t,Y_d(t))]^{-1} \tag{21}$$

$$T(x,t,Y_d(t)) \triangleq (T_1(x,t,Y_d(t)),\ldots,T_m(x,t,Y_d(t))) \tag{22}$$

where

$$T_i(x,t,Y_d(t)) \triangleq (E_i(x,y_d(t)), X_0E_i(x,y_d(t)),\ldots,X_0^{d_i}E_i(x,y_d(t))), \qquad i = 1,\ldots,m. \tag{23}$$

Then, it is not difficult to see that the system (12) with the control law:

$$u = \alpha(x,t,Y_d(t)) + \beta(x,t,Y_d(t))\overline{u}, \tag{24}$$

is transformed through the state transformation:

$$\overline{x} = T(x,t,Y_d(t)) \tag{25}$$

into the decoupled linear system:

$$\dot{\overline{x}} = \overline{A}\overline{x} + \overline{B}\overline{u}; \qquad e = \overline{C}\overline{x} \tag{26}$$

where $\overline{x}(t) \triangleq (\overline{x}_1(t),\ldots,\overline{x}_m(t))$, $\overline{x}_i(t) \triangleq (e_i(t), e_i^{(1)}(t),\ldots,e_i^{(d_i)}(t)) \in R^{d_i+1}$, $\overline{A} \triangleq \mathrm{diag}\overline{A}_i$, $\overline{B} \triangleq \mathrm{diag}\overline{b}_i$, $\overline{C} \triangleq \mathrm{diag}\overline{c}_i$, and

$$\overline{A}_i \triangleq \begin{bmatrix} 0 & & \\ \vdots & I_{di} & \\ 0 & \cdots & 0 \end{bmatrix} \in R^{(d_i+1)\times(d_i+1)}$$
$$\overline{b}_i \triangleq (0,\ldots,0,1) \in R^{d_i+1} \tag{27}$$
$$\overline{c}_i^T \triangleq (1,0,\ldots,0) \in R^{d_i+1}, \qquad i = 1,\ldots,m.$$

Hence, we can easily find the desired control law and state transformation for input–output linearization under the conditions B1–B3.

Next, we take the new input $\overline{u}$ in (24) or (26) as follows.

$$\overline{u}_i \triangleq K_i\overline{x}_i = K_iT_i(x,t,Y_d(t)), \qquad i = 1,\ldots,m. \tag{28}$$

Suppose that $K_i^T \in R^{d_i+1}$, $i = 1,\ldots,m$ are determined so that

$$\overline{A}_i + \overline{b}_iK_i, \qquad i = 1,\ldots,m, \text{ are stable matrices.} \tag{29}$$

Then, the tracking error will tend to zero since the closed-loop system given by (12), (24), and (28) has the same input–output dynamic characteristics as the stably decoupled linear system (26) with (28).

V. DESIGN OF NEW CLOS GUIDANCE LAW

In this section we design a new CLOS guidance law by applying the feedback linearizing approach described in Section IV to the tracking problem formulated in Section III. To do so, we make the following assumption in addition to the assumptions A1, A2 in Section III.

A3:

$$\mathbf{i}_M \cdot \mathbf{i}_L > 0, \qquad t \geq 0.$$

In other words, we assume that the missile flies with its X_M axis upward in the $Y_L - Z_L$ plane until target interception. This assumption is valid in the usual pursuit situations of missile and target. Note that the unit vectors $\mathbf{i}_M$, $\mathbf{i}_L$ can be represented in the inertial frame as

$$\begin{aligned} \mathbf{i}_M &= c\theta_m c\psi_m \mathbf{i}_I + c\theta_m s\psi_m \mathbf{j}_I + s\theta_m \mathbf{k}_I \\ \mathbf{i}_L &= c\gamma_t c\sigma_t \mathbf{i}_I + c\gamma_t s\sigma_t \mathbf{j}_I + s\gamma_t \mathbf{k}_I \end{aligned} \tag{30}$$

respectively. Therefore, the assumption A3 can be stated alternatively as follows

A3′:

$$c\gamma_t c\theta_m c(\psi_m - \sigma_t) + s\gamma_t s\theta_m > 0, \qquad t \geq 0.$$

Direct computation yields

$$\begin{aligned} X_1E_1 &= X_2E_1 = X_1E_2 = X_2E_2 = 0 \\ X_0E_1 &= -\dot{\sigma}_t x_1 c\sigma_t - \dot{\sigma}_t x_2 s\sigma_t - x_4 s\sigma_t + x_5 c\sigma_t \\ X_1X_0E_1 &= -s\phi_{mc} s x_8 s(x_7 - \sigma_t) + c\phi_{mc} c(x_7 - \sigma_t) \\ X_2X_0E_1 &= -c\phi_{mc} s x_8 s(x_7 - \sigma_t) - s\phi_{mc} c(x_7 - \sigma_t) \\ X_0E_2 &= (\dot{\sigma}_t s\sigma_t s\gamma_t - \dot{\gamma}_t c\gamma_t c\sigma_t)x_1 \\ &\quad - (\dot{\gamma}_t c\gamma_t s\sigma_t + \dot{\sigma}_t c\sigma_t s\gamma_t)x_2 - \dot{\gamma}_t x_3 s\gamma_t \\ &\quad - x_4 c\sigma_t s\gamma_t - x_5 s\sigma_t s\gamma_t + x_6 c\gamma_t \\ X_1X_0E_2 &= \{c\gamma_t c x_8 + s\gamma_t s x_8 c(x_7 - \sigma_t)\} s\phi_{mc} \\ &\quad + c\phi_{mc} s\gamma_t s(x_7 - \sigma_t) \\ X_2X_0E_2 &= \{c\gamma_t c x_8 + s\gamma_t s x_8 c(x_7 - \sigma_t)\} c\phi_{mc} \\ &\quad - s\phi_{mc} s\gamma_t s(x_7 - \sigma_t). \end{aligned} \tag{31}$$

By the assumption A3 or A3′,

$$\det \begin{bmatrix} X_1X_0E_1 & X_2X_0E_1 \\ X_1X_0E_2 & X_2X_0E_2 \end{bmatrix} = c\gamma_t c x_8 c(x_7 - \sigma_t) + s\gamma_t s x_8 > 0, \qquad t \geq 0. \tag{32}$$

Thus, the assumption A3 implies the conditions B1 and B3. In addition, if the flight path of target in the three-dimensional space is smooth enough so that

A4:

$$\sigma_t, \gamma_t \in C^2$$

We see that all conditions B1–B3 in Section IV are satisfied with $n = 8$, $m = 2$, $s = 2$, and $d_1 = d_2 = 1$. By (17), (18), (22), and (23), we have $d_1 = d_2 = 1$. Now, if we choose the new input $\overline{u}$ in (28) so that

$$K_1 = K_2 = [-\omega_n^2 \quad -2\zeta\omega_n] > 0, \qquad 0 < \zeta < 1, \quad \omega_n > 0 \tag{38}$$

the condition (29) is satisfied and hence the tracking

$$D^*(x,t,Y_d(t)) = \begin{bmatrix} -s\phi_{mc}sx_8s(x_7-\sigma_t) + c\phi_{mc}c(x_7-\sigma_t) & -c\phi_{mc}sx_8s(x_7-\sigma_t) - s\phi_{mc}c(x_7-\sigma_t) \\ \{c\gamma_t cx_8 + s\gamma_t sx_8c(x_7-\sigma_t)\}s\phi_{mc} & \{c\gamma_t cx_8 + s\gamma_t sx_8 c(x_7-\sigma_t)\}c\phi_{mc} \\ +c\phi_{mc}s\gamma_t s(x_7-\sigma_t) & -s\phi_{mc}s\gamma_t s(x_7-\sigma_t) \end{bmatrix} \tag{33}$$

$$\begin{aligned} T(x,t,Y_d(t)) = (&-x_1 s\sigma_t + x_2 c\sigma_t, -\dot{\sigma}_t x_1 c\sigma_t - \dot{\sigma}_t x_2 s\sigma_t - x_4 s\sigma_t + x_5 c\sigma_t, -x_1 s\gamma_t c\sigma_t - x_2 s\gamma_t s\sigma_t + x_3 c\gamma_t, \\ &(\dot{\sigma}_t s\sigma_t s\gamma_t - \dot{\gamma}_t c\gamma_t c\sigma_t)x_1 - (\dot{\gamma}_t c\gamma_t s\sigma_t + \dot{\sigma}_t c\sigma_t s\gamma_t)x_2 - \dot{\gamma}_t x_3 s\gamma_t - x_4 c\sigma_t s\gamma_t - x_5 s\sigma_t s\gamma_t + x_6 c\gamma_t). \end{aligned} \tag{34}$$

error will converge to zero. Let $K \triangleq \operatorname{diag} K_i$, $i = 1,2$ and $\overline{A}_K \triangleq \overline{A} + \overline{B}K$. Here, we have chosen the gain matrix K so that the eigenvalues of $\overline{A}_K$ are all complex. This choice has been made for simplicity of our technical developments which are presented in Section VI but does not cause any loss of generality in our results.

On the other hand, we can express R_P and $\dot{R}_P$ as

$$\begin{aligned} R_P &= x_1 c\gamma_t c\sigma_t + x_2 c\gamma_t s\sigma_t + x_3 s\gamma_t \\ \dot{R}_P &= -(\dot{\sigma}_t s\sigma_t c\gamma_t + \dot{\gamma}_t s\gamma_t c\sigma_t)x_1 - (\dot{\gamma}_t s\gamma_t s\sigma_t - \dot{\sigma}_t c\sigma_t c\gamma_t)x_2 \\ &\quad + \dot{\gamma}_t x_3 c\gamma_t + x_4 c\gamma_t c\sigma_t + x_5 c\gamma_t s\sigma_t + x_6 s\gamma_t. \end{aligned} \tag{35}$$

Using these identities with the definitions of e_1, e_2 in (6), we can write X_0E_1 and X_0E_2 in (31) as follows.

$$\begin{aligned} X_0E_1 &= -\dot{\sigma}_t R_P c\gamma_t + \dot{\sigma}_t e_2 - x_4 s\sigma_t + x_5 c\sigma_t \\ X_0E_2 &= -\dot{\sigma}_t e_1 s\gamma_t - \dot{\gamma}_t R_P - x_4 s\gamma_t c\sigma_t \\ &\quad - x_5 s\gamma_t s\sigma_t + x_6 c\gamma_t. \end{aligned}$$

By this and (34), we can express $X_0^2E_1$ and $X_0^2E_2$ as

$$\begin{aligned} X_0^2E_1 &= (2\dot{\sigma}_t\dot{\gamma}_t s\gamma_t - \ddot{\sigma}_t c\gamma_t)R_P + \dot{\sigma}_t^2 e_1 + (2\dot{\sigma}_t\dot{\gamma}_t c\gamma_t + \ddot{\sigma}_t s\gamma_t)e_2 \\ &\quad - 2\dot{\sigma}_t\dot{R}_P c\gamma_t + 2\dot{\sigma}_t\dot{e}_2 s\gamma_t + a_x(t)cx_8 s(x_7-\sigma_t) \\ X_0^2E_2 &= -(\ddot{\gamma}_t + \dot{\sigma}_t^2 s\gamma_t c\gamma_t)R_P - \ddot{\sigma}_t e_1 s\gamma_t + (\dot{\sigma}_t^2|s\gamma_t|^2 + \dot{\gamma}_t^2)e_2 \\ &\quad - 2\dot{\gamma}_t\dot{R}_P - 2\dot{\sigma}_t\dot{e}_1 s\gamma_t + \{sx_8c\gamma_t - cx_8 s\gamma_t c(x_7-\sigma_t)\} \\ &\quad \times a_x(t) - gc\gamma_t. \end{aligned} \tag{36}$$

By (19) and (36), we finally have

$$\begin{aligned} A^*(x,t,Y_d(t)) = (&(2\dot{\sigma}_t\dot{\gamma}_t s\gamma_t - \ddot{\sigma}_t c\gamma_t)R_P + \dot{\sigma}_t^2 e_1 \\ &+ (2\dot{\sigma}_t\dot{\gamma}_t c\gamma_t + \ddot{\sigma}_t s\gamma_t)e_2 \\ &- 2\dot{\sigma}_t\dot{R}_P c\gamma_t + 2\dot{\sigma}_t\dot{e}_2 s\gamma_t + a_x(t)cx_8s(x_7-\sigma_t), \\ &- (\ddot{\gamma}_t + \dot{\sigma}_t^2 s\gamma_t c\gamma_t)R_P - \ddot{\sigma}_t e_1 s\gamma_t + (\dot{\sigma}_t^2|s\gamma_t|^2 + \dot{\gamma}_t^2)e_2 \\ &- 2\dot{\gamma}_t\dot{R}_P - 2\dot{\sigma}_t\dot{e}_1 s\gamma_t + \{sx_8c\gamma_t - cx_8s\gamma_t c(x_7-\sigma_t)\} \\ &\times a_x(t) - gc\gamma_t). \end{aligned} \tag{37}$$

Using (33), (34), and (37), we can construct the control law of the form (24) which transforms the system (7) into the decoupled system (26) with $m = 2$ and

The final form of our new CLOS guidance law is given by

$$\begin{aligned} u &\triangleq [D^*(x,t,Y_d(t))]^{-1}\{KT(x,t,Y_d(t)) - A^*(x,t,Y_d(t))\} \\ &\triangleq -\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \begin{bmatrix} p_1 + q_1 \\ p_2 + q_2 \end{bmatrix} \Big/ a \end{aligned} \tag{39}$$

where

$$\begin{aligned} a &\triangleq c\gamma_t c\theta_m c(\psi_m - \sigma_t) + s\gamma_t s\theta_m \\ b_1 &\triangleq \{c\gamma_t c\theta_m + s\gamma_t s\theta_m c(\psi_m - \sigma_t)\}c\phi_{mc} - s\phi_{mc}s\gamma_t s(\psi_m - \sigma_t) \\ b_2 &\triangleq c\phi_{mc}s\theta_m s(\psi_m - \sigma_t) + s\phi_{mc}c(\psi_m - \sigma_t) \\ b_3 &\triangleq -\{c\gamma_t c\theta_m + s\gamma_t s\theta_m c(\psi_m - \sigma_t)\}s\phi_{mc} - c\phi_{mc}s\gamma_t s(\psi_m - \sigma_t) \\ b_4 &\triangleq -s\phi_{mc}s\theta_m s(\psi_m - \sigma_t) + c\phi_{mc}c(\psi_m - \sigma_t) \\ p_1 &\triangleq \omega_n^2 e_1 + 2\zeta\omega_n\dot{e}_1 + (2\dot{\sigma}_t\dot{\gamma}_t s\gamma_t - \ddot{\sigma}_t c\gamma_t)R_P - 2\dot{\sigma}_t\dot{R}_P c\gamma_t \\ &\quad + \{c\theta_m s(\psi_m - \sigma_t)\}(T - D)/M \\ p_2 &\triangleq \omega_n^2 e_2 + 2\zeta\omega_n\dot{e}_2 - (\ddot{\gamma}_t + \dot{\sigma}_t^2 s\gamma_t c\gamma_t)R_P - 2\dot{\gamma}_t\dot{R}_P \\ &\quad + \{s\theta_m c\gamma_t - c\theta_m s\gamma_t c(\psi_m - \sigma_t)\}(T - D)/M - gc\gamma_t \\ q_1 &\triangleq \dot{\sigma}_t^2 e_1 + (2\dot{\sigma}_t\dot{\gamma}_t c\gamma_t + \ddot{\sigma}_t s\gamma_t)e_2 + 2\dot{\sigma}_t\dot{e}_2 s\gamma_t \\ q_2 &\triangleq -\ddot{\sigma}_t e_1 s\gamma_t + (\dot{\sigma}_t^2|s\gamma_t|^2 + \dot{\gamma}_t^2)e_2 - 2\dot{\sigma}_t\dot{e}_1 s\gamma_t. \end{aligned} \tag{40}$$

Now, it should be clear that the missile guided by this guidance law will always hit any randomly maneuvering target if the gains in (38) are chosen to give a sufficiently fast convergence rate in the tracking error.

Most of the guidance informations required by our guidance law (39) can be acquired directly from the ground tracker and on-board inertial navigation unit (INU). Specifically, they are R_m, σ_t, γ_t, $\dot{\sigma}_t$, $\dot{\gamma}_t$, $\Delta\sigma$, $\Delta\gamma$, ψ_m, and θ_m. Since R_P, e_1 and e_2 cannot be measured directly, these quantities ought to be computed indirectly using the polar position data of the missile available from the ground tracker as follows.

$$\begin{aligned} R_P &= R_m c(\Delta\gamma + \gamma_t) c\gamma_t c\Delta\sigma + R_m s(\Delta\gamma + \gamma_t) s\gamma_t \\ e_1 &= R_m c(\Delta\gamma + \gamma_t) s\Delta\sigma \\ e_2 &= R_m s(\Delta\gamma + \gamma_t) c\gamma_t - R_m c(\Delta\gamma + \gamma_t) s\gamma_t c\Delta\sigma. \end{aligned} \tag{41}$$

Information of the time derivatives $\dot{R}_P$, $\dot{e}_1$, $\dot{e}_2$, $\ddot{\sigma}_t$, and $\ddot{\gamma}_t$ can be estimated via band-limited differentiators or Kalman filters. Also the time history of $(T - D)/M$ can be estimated from the experimental aero-data.

Now, we give some comments on the earlier results closely related to ours. Our guidance law involves some terms that are, in the form, similar to feedforward acceleration and error compensation acceleration called in the earlier results [5, 21, 22]. The feedforward acceleration term was to make the missile chase the LOS rotation, while the error compensation acceleration term was to null deviation of the missile from the LOS to target. In the earlier results, these terms were calculated under some restrictive assumptions. In [5, 21], it was assumed that the missile is on the LOS to target and its velocity vector lies on the flyplane, which is the imaginary plane spanned by the target velocity and the LOS to target. In [22], the lateral acceleration of the missile projection point onto the LOS to target was adopted as the feedforward acceleration term. Therefore, their guidance laws may undergo some performance degradation in the case when such restrictive assumptions are not valid.

Note that these assumptions are not required in the derivation of our guidance law in (39). Recall that the terms $-[D^*]^{-1}A^*$, $[D^*]^{-1}KT$ in our guidance law in (39) stand for the lateral missile accelerations required to achieve, respectively, $\ddot{e}_1 = \ddot{e}_2 = 0$ and $\lim_{t\to\infty}|e| = 0$. In this context, we can view the terms $-[D^*]^{-1}A^*$, $[D^*]^{-1}KT$ as the precise expressions of the so-called feedforward acceleration and error compensation acceleration, respectively, for the case of general pursuit situations. Hence, our result, in some sense, completes the earlier works in [5, 21, 22]. The recently developed nonlinear control technique has proven to be a useful tool in doing so.

In the practical viewpoint, the new CLOS guidance law in (39) is computationally complex. Moreover, the promised performance of our guidance law cannot be fully achieved due to modeling errors such as the ground tracker and autopilot dynamics. Therefore, it may be more practical to find a guidance law which requires less computation but whose guidance performance is still satisfactory. In the next section, we explore this problem.

VI. SIMPLIFICATION OF NEW CLOS GUIDANCE LAW

In this section, we attempt to simplify the new CLOS guidance law in (39) in order to reduce computational burden without significant performance degradation. For this aim, we drop out some terms (specifically, q_1 and q_2 in (40)) in A^* to obtain the following simplified CLOS guidance law:

$$\begin{aligned} u &\triangleq [D^*(x,t,Y_d(t))]^{-1}\{KT(x,t,Y_d(t)) - \hat{A}^*(x,t,Y_d(t))\} \\ &\triangleq -\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \Big/ a. \end{aligned} \tag{42}$$

Comparing two guidance laws in (39) and (42), we can see that such simplification reduces the computational burden almost by half.

Let $\Delta A^* \triangleq A^* - \hat{A}^*$. Then, we see that ΔA^* has the form:

$$\Delta A^* = \Delta K T \tag{43}$$

where

$$\Delta K \triangleq \begin{bmatrix} \dot{\sigma}_t^2 & 0 & 2\dot{\sigma}_t\dot{\gamma}_t c\gamma_t + \ddot{\sigma}_t s\gamma_t & 2\dot{\sigma}_t s\gamma_t \\ -\ddot{\sigma}_t s\gamma_t & -2\dot{\sigma}_t s\gamma_t & \dot{\sigma}_t^2 |s\gamma_t|^2 + \dot{\gamma}_t^2 & 0 \end{bmatrix}. \tag{44}$$

Note that the velocity and acceleration vectors of the target can be represented as follows [7].

$$\begin{aligned} \mathbf{v}_t &= \dot{R}_t \mathbf{i}_L + R_t\dot{\sigma}_t c\gamma_t \mathbf{j}_L + R_t\dot{\gamma}_t \mathbf{k}_L \\ \mathbf{a}_t &= (\ddot{R}_t - R_t\dot{\gamma}_t^2 - R_t\dot{\sigma}_t^2|c\gamma_t|^2)\mathbf{i}_L \\ &\quad + (R_t\ddot{\sigma}_t c\gamma_t - 2R_t\dot{\sigma}_t\dot{\gamma}_t s\gamma_t + 2\dot{R}_t\dot{\sigma}_t c\gamma_t)\mathbf{j}_L \\ &\quad + (2\dot{R}_t\dot{\gamma}_t + R_t\ddot{\gamma}_t + R_t\dot{\sigma}_t^2 s\gamma_t c\gamma_t)\mathbf{k}_L. \end{aligned} \tag{45}$$

Using this fact and the inequality

$$||\Delta K|| \leq \hat{|}\Delta K\hat{|} \tag{46}$$

we can derive a rough but useful bound of $||\Delta K||$.

$$\begin{aligned} ||\Delta K|| \leq \{&(10|c\gamma_t|^{-4} + 16|c\gamma_t|^{-3} + 18|c\gamma_t|^{-2} + 8|c\gamma_t|^{-1} + 3)v_t^4/R_t^4 \\ &+ (8|c\gamma_t|^{-3} + 8|c\gamma_t|^{-2} + 4|c\gamma_t|^{-1})a_t v_t^2/R_t^3 \\ &+ 2|c\gamma_t|^{-2}a_t^2/R_t^2 + 8|c\gamma_t|^{-2}v_t^2/R_t^2\}^{1/2}. \end{aligned} \tag{47}$$

In the usual pursuit situations, v_t and a_t are upper-bounded whereas R_t and $|c\gamma_t|$ are lower-bounded. Therefore, the right-hand side (RHS) term of the inequality in (47) is upper-bounded. Thus, we can assume that

A5:

There exists a constant $\delta > 0$ such that $\|\Delta K\| \leq \delta$,

$$t \geq 0. \qquad (48)$$

Now, we show that the simplified guidance law in (42) can give almost the same guidance performance as the original one in (39) provided that ΔK is sufficiently small, specifically,

$$\delta < 0.5. \qquad (49)$$

Unfortunately, it seems difficult to describe the largest ranges of R_t, v_t, a_t and γ_t so that the condition in (49) holds. However, using the rough bound in (47), we can easily show that if $R_t \geq 4$ km, $a_t \leq 10\ g$, $v_t \leq 400$ m/s, and $0 \leq \gamma_t \leq 50$ deg, then $\|\Delta K\| < 0.5$ and hence the condition (49) is satisfied.

Choose ζ, ω_n in (38) so that

$$\zeta(1-\zeta^2)^{1/2} > \delta$$
$$\omega_n^2 > \delta(1-\zeta^2)^{1/2}/\zeta + \delta\zeta^2/\{\zeta(1-\zeta^2)^{1/2} - \delta\}. \qquad (50)$$

Note that such choice is always possible if the condition in (49) is satisfied. Since $\overline{A}_K$ has simple structure, it can be transformed to a real canonical form. To see this, take

$$P_K \triangleq \begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix}; \qquad P_0 \triangleq \begin{bmatrix} 1 & 0 \\ -\zeta\omega_n & -\omega_n(1-\zeta^2)^{1/2} \end{bmatrix}. \qquad (51)$$

Then,

$$\hat{A}_K \triangleq P_K^{-1}\overline{A}_K P_K = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix}$$
$$\Lambda \triangleq \begin{bmatrix} -\zeta\omega_n & -\omega_n(1-\zeta^2)^{1/2} \\ \omega_n(1-\zeta^2)^{1/2} & -\zeta\omega_n \end{bmatrix}. \qquad (52)$$

Furthermore,

$$\|P_K\| = [(\omega_n^2+1) + \{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2\}^{1/2}]^{1/2}/\sqrt{2}$$
$$\|P_K^{-1}\| = \|P_K\|/\{\omega_n(1-\zeta^2)^{1/2}\}. \qquad (53)$$

Choose a Lyapunov-like function V by

$$V(v(t)) = |v(t)|^2/2; \qquad v(t) = P_K^{-1}T(x(t),t,Y_d(t)). \qquad (54)$$

Since T, x, Y_d are all C^1, we can take the total time derivative of $\overline{x}(t) = T(x(t),t,Y_d(t))$ along the solution trajectory of the closed-loop system (7) with the simplified CLOS guidance law in (42). Then, we have

$$\dot{\overline{x}} = (\overline{A}_K + \overline{B}\Delta K)\overline{x}. \qquad (55)$$

By (48) and (51)–(55) with the fact that $\|\overline{B}\| = 1$,

$$\begin{aligned} \dot{V} &= v^T P_K^{-1}\dot{\overline{x}} \\ &= v^T P_K^{-1}\overline{A}_K P_K v + v^T P_K^{-1}\overline{B}\Delta K P_K v \\ &\leq v^T \hat{A}_K v + \delta\|P_K^{-1}\|\,\|P_K\|\,\|\overline{B}\|\,|v|^2 \\ &\leq -(\zeta\omega_n - \delta\|P_K^{-1}\|\,\|P_K\|)|v|^2 \\ &= -2\mu V \end{aligned} \qquad (56)$$

where

$$\mu \triangleq \zeta\omega_n - \delta[(\omega_n^2+1) + \{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2\}^{1/2}]/\{2\omega_n(1-\zeta^2)^{1/2}\}. \qquad (57)$$

This implies that

$$|v(t)| \leq |v(0)|e^{-\mu t}, \qquad t \geq 0. \qquad (58)$$

This with the fact that

$$e = \overline{C}P_K v; \qquad \overline{C}P_K = \overline{C}; \qquad \|\overline{C}\| = 1 \qquad (59)$$

yields the desired result:

$$|e(t)| \leq \rho e^{-\mu t}, \qquad t \geq 0 \qquad (60)$$

where

$$\rho \triangleq |T(x(0),0,Y_d(0))|[(\omega_n^2+1) + \{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2\}^{1/2}]^{1/2}/[\omega_n\{2(1-\zeta^2)\}^{1/2}]. \qquad (61)$$

Now, what remains is to show that

$$\mu > 0. \qquad (62)$$

For this, observe that (38) and (50) imply that

$$\begin{aligned} &\{(\omega_n^2+1) - 2\delta(1-\zeta^2)^{1/2}/\zeta\}^2 - \{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2\} \\ &= 4[\{\omega_n^2 - \delta(1-\zeta^2)^{1/2}/\zeta\}\{\zeta(1-\zeta^2)^{1/2} - \delta\} - \delta\zeta^2] \\ &\times (1-\zeta^2)^{1/2}/\zeta > 0. \end{aligned} \qquad (63)$$

On the other hand, simple calculation using the second inequality in (50) yields the following inequality

$$\omega_n^2 + 1 > 2\delta(1-\zeta^2)^{1/2}/\zeta. \qquad (64)$$

By (63) and (64), we finally have

$$\zeta[(\omega_n^2+1) - \{(\omega_n^2-1)^2 + 4\zeta^2\omega_n^2\}^{1/2}] - 2\delta(1-\zeta^2)^{1/2} > 0. \qquad (65)$$

Then, it is easy to see that (65) assures (62).

Thus, we have shown that, if the target maneuvers so that the condition in (49) holds, then the missile guided by the simplified CLOS guidance law in (42) will hit the target.

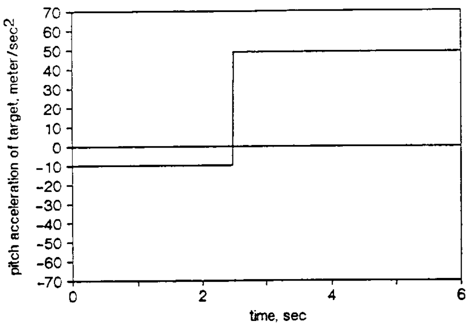

Fig. 3. Profile of target pitch acceleration.

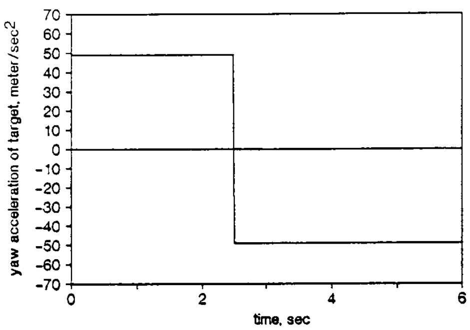

Fig. 4. Profile of target yaw acceleration.

VII. SIMULATION RESULTS

By simulations, we verify guidance performance of our new CLOS guidance law and its simplified version for short-range air defense scenario. We also investigate the effect of the autopilot and ground tracker dynamics on guidance performance.

In modeling the target motion for simulation study, we assume that the target produces no axial acceleration and roll motion. Then, the simplified dynamics of target motion can be represented in the inertial frame as follows.

$$\begin{aligned} \ddot{x}_t &= -a_{ty} s\psi_t - a_{tz} s\theta_t c\psi_t \\ \ddot{y}_t &= a_{ty} c\psi_t - a_{tz} s\theta_t s\psi_t \\ \ddot{z}_t &= a_{tz} c\theta_t - g \\ \dot{\psi}_t &= a_{ty}/(v_t c\theta_t) \\ \dot{\theta}_t &= (a_{tz} - g c\theta_t)/v_t \end{aligned} \tag{66}$$

where v_t is given by

$$v_t \triangleq (\dot{x}_t^2 + \dot{y}_t^2 + \dot{z}_t^2)^{1/2}. \tag{67}$$

As can be seen from Figs. 3 and 4, we assume that the target maneuvers with $a_{ty} = 5g$, $a_{tz} = -g$ for the first 2.5 s and then with $a_{ty} = -5g$, $a_{tz} = 5g$ until interception.

Two engagement scenarios are considered. In the first case, the target and missile are initially,

TABLE I
Simulation Data Used Commonly in Two Engagement Scenarios

ζ, ω_n	$1/\sqrt{2}$, $2\sqrt{2}$
(T-D)/M	{ 340 m/s², 0<t≤2 -44.1 m/s², t>2
ϕ_{mc}	0 deg
guidance command frequency	50 Hz
autopilot damping ratio	0.6 _
autopilot natural frequency	6π rad/s

Note: Roman symbols here correspond to italic symbols in text.

TABLE II
Initial Conditions of Target and Missile Chosen for Simulation Study

state \ case	Engagement scenario #1	Engagement scenario #2
$x_t(0)$, $y_t(0)$, $z_t(0)$ [m]	2500, 5361.9, 1000	400, 190, 2967.14
$\dot{x}_t(0)$, $\dot{y}_t(0)$, $\dot{z}_t(0)$ [m/s]	0, -340, 0	-340, 0, 0
$\psi_t(0)$, $\theta_t(0)$ [deg]	-90, 0	180, 0
$x_m(0)$, $y_m(0)$, $z_m(0)$ [m]	14.32, 39.34, 3.36	8.45, 4.96, 40.84
$\dot{x}_m(0)$, $\dot{y}_m(0)$, $\dot{z}_m(0)$ [m/s]	70.84, 151.92, 28.32	22.67, 10.77, 168.14
$\psi_m(0)$, $\theta_m(0)$ [deg]	65, 9.59	25.41, 81.51
$\Delta\sigma(0)$, $\Delta\gamma(0)$ [deg]	5, -5	5, -5

respectively, 6 km and 42 m away from the location of the ground tracker. In the second case, the initial distances of the target and missile are, respectively, 3 km and 42 m away from the location of the ground tracker. The detailed data used in the simulations for both cases are listed in Tables I and II.

First, we have performed the computer simulation with the first engagement scenario. The simulation results of the closed-loop system (1), (2), and (66) with the new CLOS guidance law (39) are shown in Figs. 6 and 7. From Fig. 6, we see that our new CLOS guidance law (39) drives the miss distance to zero. The tracking error converges to zero with decoupled linear dynamic characteristics. We have also investigated guidance performance of the simplified CLOS guidance law in (42). However, the simulation results with the simplified CLOS guidance law in (42) were nearly the same as those with the original one in (39) and hence are omitted. This is because the magnitudes of the neglected terms are relatively very small over the whole flight time, as can be seen from Fig. 8. Note that the condition in (49) is satisfied in the case of the first engagement scenario.

Next, we have investigated the significance of the condition in (49) by using the second engagement scenario. The simulation results for the second engagement scenario with the simplified guidance law in (42) are shown in Figs. 10 and 11. As can be seen from Fig. 9, the condition in (49) is violated during the initial phase of guidance. However, the simplified guidance law still drives the miss distance to zero since the condition in (49) holds for the final phase of guidance until interception. The time history of the tracking error in Fig. 10 also suggests that

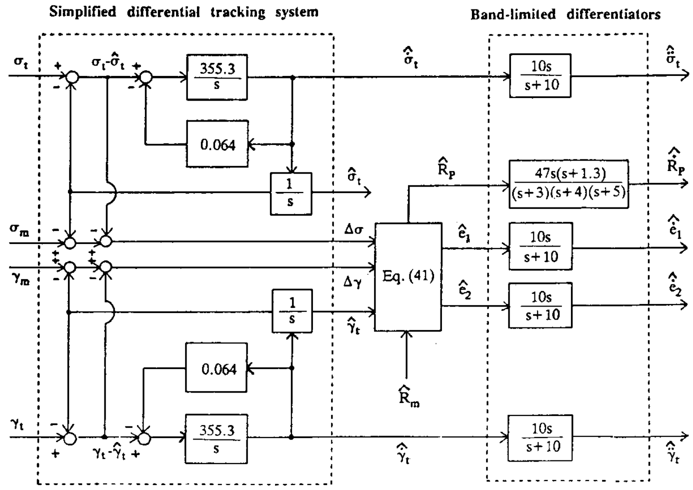

Fig. 5. Block diagram representation of estimation algorithm for guidance information.

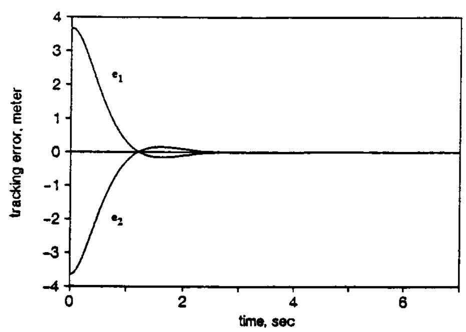

Fig. 6. Time history of tracking error. (Engagement scenario 1 with new CLOS guidance law.)

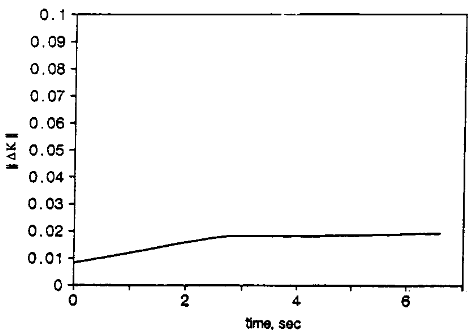

Fig. 8. Time history of $||\Delta K||$ (engagement scenario 1).

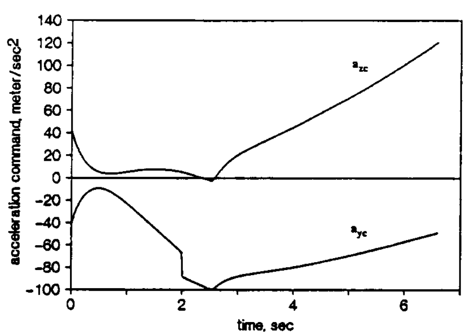

Fig. 7. Time history of acceleration command. (Engagement scenario 1 with new CLOS guidance law.)

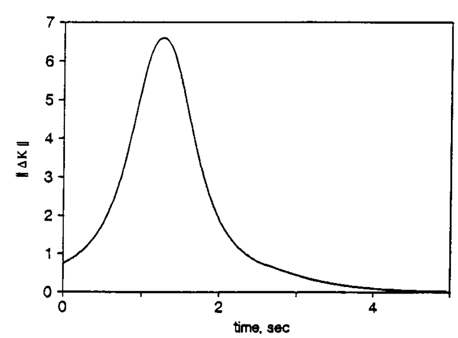

Fig. 9. Time history of $||\Delta K||$ (engagement scenario 2).

the simplified guidance law may not guarantee zero miss distance when the condition in (49) continues to be violated until interception. However, it should be noted that (49) is only a sufficient condition for interception. Moreover, as has been discussed in Section VI, the condition in (49) holds in the usual engagement situation. In fact, the second engagement scenario represents a special pursuit situation, in which the target passes over the ground tracker.

Next, we have included both of the pitch and yaw autopilot dynamics as second-order time-invariant linear systems and the ground tracker as a simplified differential tracking system with damping ratio 0.6 and natural frequency 6π rad/s (see Fig. 5). The ground

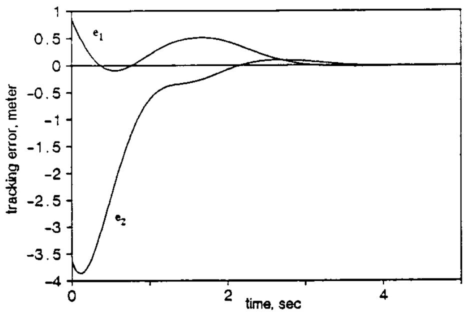

Fig. 10. Time history of tracking error. (Engagement scenario 2 with simplified CLOS guidance law.)

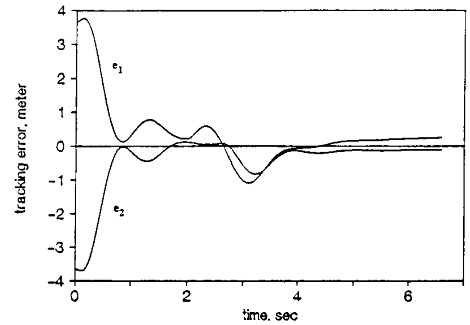

Fig. 12. Time history of tracking error. (Engagement scenario 1 with simplified CLOS guidance law, ground tracker and autopilot dynamics, and measurement and estimation errors.)

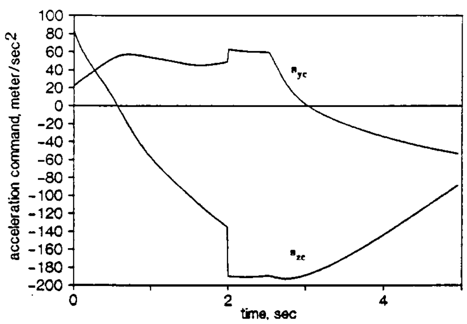

Fig. 11. Time history of acceleration command. (Engagement scenario 2 with simplified CLOS guidance law.)

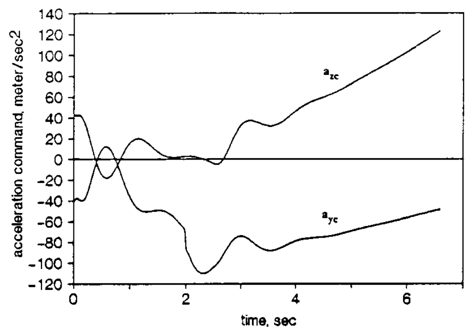

Fig. 13. Time history of acceleration command. (Engagement scenario 1 with simplified CLOS guidance law, ground tracker and autopilot dynamics, and measurement and estimation errors.)

tracker provides the estimated values of σ_t, γ_t, $\dot{\sigma}_t$, and $\dot{\gamma}_t$ as well as the measurement data of $\Delta\sigma$ and $\Delta\gamma$. In what follows, the estimated value of a variable is distinguished from its true value by inserting the upper $\wedge$ to the corresponding variable. We assume that the data of ψ_m, θ_m, and R_m are available. The experimental data of $(T - D)/M$ are assumed to involve a 20 percent scale factor error. The simulation results for such a nonideal case with the simplified guidance law in (42) are shown in Figs. 12 and 13.

As is shown in Fig. 5, the estimated values of the time-derivatives $\dot{R}_P$, $\dot{e}_1$, $\dot{e}_2$, $\dot{\sigma}_t$, and $\dot{\gamma}_t$ are obtained from those of R_P, e_1, e_2, $\dot{\sigma}_t$, and $\dot{\gamma}_t$ via conventional band-limited differentiation. Particularly for the estimation of $\dot{R}_P$, we have designed a differentiator of type 2 since the time history of R_P is typically almost a ramp function. The modern stochastic approach to the estimation of target motion would be construction of a Kalman filter. Because of limited space, however, we do not pursue this issue further here.

The simulation result in Fig. 12 shows that the ground tracker and autopilot dynamics as well as the estimation and measurement errors cause some performance degradation. At 6.58 s, the guidance is ended with miss distance 0.28 m.

VIII. CONCLUSION

We have presented a novel approach to the three-dimensional CLOS guidance problem. We have shown that the CLOS guidance problem can be converted to a nonlinear tracking problem so that the recently developed approach to robust tracking in nonlinear systems can be applied effectively. As is shown through simulations, however, fast ground tracker and autopilot dynamics as well as small measurement and estimation errors are crucially important for good performance of our CLOS guidance laws.

Our work differs from the earlier one, mainly in that nonlinearity of the guidance mechanism is fully taken into account in the design process of CLOS guidance law. Our result can be easily extended to bank-to-turn (BTT) missiles. Like the results in [4, 11, 14], our result can be generalized so as to take into account autopilot dynamics. However, the resulting guidance law gets very complex and its performance may be inevitably affected by variation of autopilot parameters. Therefore, in the case of fast autopilot and ground tracker dynamics, we would rather neglect these dynamics to obtain a guidance law which is robust, but at the cost of degraded performance. On the other hand, in order to reduce the effect of measurement and estimation errors on guidance performance, we may have to incorporate the well-known stochastic estimation methods into our guidance law, as in the works of [4, 11, 14].

ACKNOWLEDGMENT

The authors would like to thank Dr. T.-L. Song and Mr. J.-Y. Ahn at the Agency for Defense Development and Professor J. G. Lee at Seoul National University for their helpful suggestions and comments during this work. The authors also wish to thank Dr. J.-S. Hur for his extensive survey on the conventional CLOS schemes performed for the basis of this work.

REFERENCES

[1] Byrn, C. I., and Isidori, A. (1985)
Global feedback stabilization of nonlinear systems.
In *IEEE Decision and Control Conference Proceedings*, **24** (Dec. 1985).

[2] Carrington, C. K., and Junkins, J. L. (1984)
Nonlinear feedback control of spacecraft slew maneuvers.
The Journal of the Astronautical Sciences, **32** (Jan.–Mar. 1984), 29–45.

[3] Dwyer, T. A. W., III (1984)
Exact nonlinear control of large angle rotational maneuvers.
IEEE Transactions on Automatic Control, **AC-29**, 9 (Sept. 1984), 769–774.

[4] Fleming, R. J., and Irwin, G. W. (1987)
Filter controllers for bank-to-turn CLOS guidance.
IEE Proceedings, **134**, Pt. D, 1 (Jan. 1987), 17–25.

[5] Garnell, P., and East, D. J. (1977)
Guided Weapon Control Systems.
Oxford: Pergamon Press, 1977, 134–167.

[6] Gilbert, E. G., and Ha, I. J. (1984)
An approach to nonlinear feedback control with applications to robotics.
IEEE Transactions on Systems, Man and Cybernetics, **SMC-14** (1984), 879–884.

[7] Greenwood, D. T. (1980)
Principles of Dynamics.
Englewood Cliffs, NJ: Prentice-Hall, 1980, 37–38.

[8] Ha, I. J. (1988)
The standard decomposed system and noninteracting feedback control of nonlinear systems.
SIAM Journal of Control and Optimization, **26**, 5 (Sept. 1988), 1235–1249.

[9] Ha, I. J., and Gilbert, E. G. (1987)
Robust tracking in nonlinear systems.
IEEE Transactions on Automatic Control, **AC-32**, 9 (Sept. 1987), 763–771.

[10] Hunt, L. R., Luksic, M., and Su, R. (1986)
Exact linearization of input–output systems.
International Journal of Control, **43**, 1 (1986), 247–255.

[11] Irwin, G. W. (1986)
Design of controllers for bank-to-turn CLOS guidance using optimal control.
In *Proceedings of the American Control Conference*, June 1986, 1143–1148.

[12] Isidori, A. (1985)
The matching of a prescribed linear input–output behavior in a nonlinear system.
IEEE Transactions on Automatic Control, **AC-30**, 3 (Mar. 1985), 258–265.

[13] Isidori, A., Krener, A. J., Gori-Giorgi, C., and Monaco, S. (1981)
Nonlinear decoupling via feedback: a differential geometric approach.
IEEE Transactions on Automatic Control, **AC-26** (1981), 331–345.

[14] Kain, J. E., and Yost, D. J. (1977)
Command to line-of-sight guidance: a stochastic optimal control problem.
Journal of Spacecraft and Rockets, **14**, 7 (July 1977), 438–444.

[15] Lane, S. H., and Stengel, R. F. (1988)
Flight control design using nonlinear inverse dynamics.
Automatica, **24** (1988), 471–484.

[16] Lee, H. G., Arapostathis, A., and Marcus, S. I. (1987)
Linearization of discrete time systems.
International Journal of Control, **45** (1987), 1803–1822.

[17] Menon, P. K. A., Badgett, M. E., Walker, R. A., and Duke, E. L. (1987)
Nonlinear flight test trajectory controllers for aircraft.
Journal of Guidance, Control, Dynamics, **10**, 1 (1987), 67–72.

[18] Meyer, G., Su, R., and Hunt, L. R. (1984)
Application of nonlinear transformations to automatic flight control.
Automatica, **20**, 1 (1984), 103–107.

[19] Monaco, S., and Stornelli, S. (1985)
A nonlinear attitude control law for a satellite with flexible appendages.
In *IEEE Decision and Control Conference Proceedings*, **24** (Dec. 1985), 1654–1659.

[20] Pastrick, H. L., Seltzer, S. M., and Warren, M. E. (1981)
Guidance laws for short-range tactical missiles.
Journal of Guidance and Control, **4**, 2 (Mar.–Apr. 1981), 98–108.

[21] Shepherd, J. T., Stollery, J. L., and Lipscombe, J. M. (1979)
The effect of guidance and control on missile design.
AGARD Short Course, Rome, Italy, Oct. 1979.

[22] Siegel, J., and Lee, J. G. (1978)
Evaluation of command to line-of-sight guidance for medium range missiles.
Final Technical Report, TR-1053-2, The Analytic Sciences Corporation, June 1978.

[23] Singh, S. N. (1989)
Decoupled ultimate boundedness control of systems and large aircraft maneuver.
IEEE Transactions on Aerospace and Electronic Systems, **25**, 5 (Sept. 1989), 677–688.

[24] Singh, S. N., and Iyer, A. (1989)
Nonlinear decoupling sliding mode control and attitude control of spacecraft.
IEEE Transactions on Aerospace and Electronic Systems, **25**, 5 (Sept. 1989), 621–633.

[25] Tahk, M., Briggs, M. M., and Menon, P. K. A. (1988)
Applications of plant inversion via state feedback to missile autopilot design.
In *IEEE Decision and Control Conference Proceedings*, Austin, TX, Dec. 1988.

In-Joong Ha received the B.S. and M.S. degrees in electronics engineering from Seoul National University, Seoul, Korea, in 1973 and 1980, respectively, and the Ph.D. degree in computer, information, and control engineering (CICE) from the University of Michigan, Ann Arbor, in 1985.

He is presently an Associate Professor in the Department of Control and Instrumentation Engineering, Seoul National University, Seoul, Korea. From 1973 to 1981, he worked in the area of missile guidance and control at the Agency for Defense Development in Daejeon, Korea. From 1982 to 1985, he was a Research Assistant at the Center for Research on Integration Manufacturing, University of Michigan, Ann Arbor. From 1985 to 1986, he worked as a Senior Research Engineer at General Motors Research Laboratories, Warren, MI. His fields of interest are nonlinear control theory and its application to robotics, electric machines, and missiles.

Dr. Ha was the recipient of the 1985 Outstanding Achievement Award in the CICE program.

Song Chong was born in Seoul, Korea, on Oct. 7, 1964. He received the B.S. and M.S. degrees in control and instrumentation engineering from Seoul National University, Seoul, Korea, in 1988 and 1990, respectively.

He is presently working at the Robotics and Intelligent Systems Laboratory, Seoul National University. His current research interests are nonlinear control theory and its application to missiles.