First-order rate-based flow control with dynamic queue threshold for high-speed wide-area ATM networks

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ABSTRACT

In this paper we present a new rate-based flow control scheme for ATM ABR services and analyze its performance. The proposed algorithm, which we refer to as *First-order Rate-based Flow Control* (FRFC) is the most simple form of queue-length-based flow control. The asymptotic stability, the steady-state throughput, queue length and fairness, and the transient behavior are analyzed for the case of multiple connections with diverse round-trip delays. We also consider a novel approach to dynamically adjust a queue threshold in the FRFC according to the changes in the available bandwidth, and the arrival and departure of connections. Simulations show that the simple FRFC with dynamic queue threshold (DQT) effectively maintains high throughput, small loss and a desired fairness in these dynamic environments and is a promising solution for ABR flow control in ATM networks.

Keywords: ABR flow control, stability, fairness, transient performance, ATM networks

1. INTRODUCTION

Recently there has been a great interest in feedback-based flow control for high-speed wide-area networking. In particular, a *rate-based approach* has been studied extensively[2–8] and adopted by the ATM Forum as the standard for the flow control of the Available Bit Rate (ABR) service[1,2].

The rate-based flow control problem in high-speed wide-area networks can be stated as follows. Consider a network with a single bottleneck link as depicted in Figure 1. The geographically distributed sources transmit data into the bottleneck node in their path at the rate allocated by the node. In reality, the bottleneck can be any node in the network and for simplicity, we consider only a single link in the network as a bottleneck. The switch computes the rates that will be allocated to the sources. In the queue-length-based rate control that we consider in this paper, the rates are computed based on a certain function of the difference between the observed queue length and a queue threshold. In this type of approach, a certain fairness in rate allocation among users is accomplished as a consequence of the queue-length control. Examples of this type can be found in the literature[4–7]. The other type of rate-based flow control[8,9] is to compute directly rate allocations in a way that a certain fairness property is satisfied. Typically, in this latter approach, the queue length is not explicitly controlled. Communication between the node and the sources is done via special cells that are embedded into the individual data streams. It is well understood that the large bandwidth-delay product involved in the problem can cause a loose control with non-negligible loss and link under-utilization.

There are important criteria in the design of high-performance ABR flow control algorithms. In the following, we summarize the criteria in consideration in this paper.

- Maximal link utilization and small cell loss, and consequently maximal throughput in steady-state.
- Stability (preferably asymptotic stability) of the steady-state solution for the case of multiple virtual circuits (VCs) with long and diverse round-trip delays.
- Fair bandwidth allocation among ABR streams; guarantee of standard fairness criteria such as *MAX-MIN fair share.*[1]

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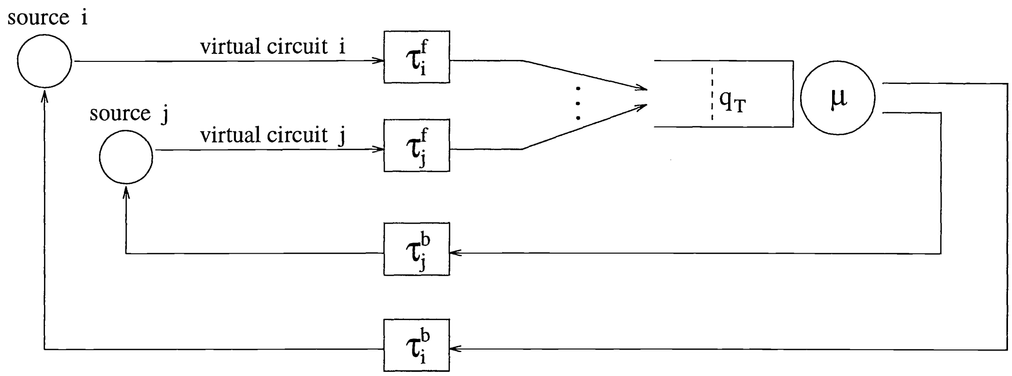

Figure 1: Network model with a single bottleneck link

- Good transient performance such as fast and uniform convergence irrespective of number of active VCs.
- Adaptability to the changes in the operational environment, for instance, changes in available bandwidth and the number of active VCs.
- Simplicity in implementation.

To meet these criteria, we introduce a new rate-based flow control algorithm which we refer to as *First-Order Rate-Based Flow Control* (FRFC). The FRFC is a queue-length-based flow control algorithm where the rate allocated to each ABR user is the difference between observed network queue length and queue threshold, multiplied by a control gain. In contrast, most other existing queue-length-based algorithms[4−7,12] control the derivative of the rate as a certain function of queue length and thus is viewed as a second-order rate control. The analysis in the paper will show that the proposed algorithm can make the network queue and the user rates asymptotically stable even for the case of multiple VCs with long and diverse delays, if the control gains are properly chosen. Also, it will be shown that in the steady state the rate allocation satisfies the MAX-MIN fairness criterion. To further improve the performance of the FRFC algorithm in dynamic environments where available bandwidth varies and VCs frequently join and leave, we consider an approach to dynamically change the queue threshold in the FRFC whenever the changes in the available bandwidth and the set of active VCs are detected. The simulation study shows that the simple FRFC with dynamic queue threshold (DQT) effectively maintains high throughput, small loss, and MAX-MIN fairness in rate allocation in such dynamic environments.

The paper is organized as follows. Section 2 describes the network model and the FRFC algorithm. In Section 3, the asymptotic stability and steady-state response of the network with FRFC are addressed. Transient behavior is considered in Section 4 and the dynamic queue threshold is introduced in Section 5. In Section 6 simulations are presented, and the conclusion appears in Section 7.

2. MODEL AND CONTROL ALGORITHM

The assumptions employed for the analysis of the FRFC algorithm are as follows and are fairly standard[5−7]:

A.1. The traffic is viewed as a deterministic fluid flow and the network queueing process and the feedback control is continuous in time. This assumption enables us to model the closed-loop system by a differential equation.

A.2. The round-trip-time, τ_i, of virtual circuit i is the sum of forward-path delay, τ_i^f, and the backward-path delay, τ_i^b, which consists of propagation, queueing, transmission and processing times. We assume that τ_i is a constant. This is a reasonable assumption in a wide-area network where propagation delays are expected to dominate.

A.3. The sources are *persistent* until the system reaches steady state. By the term persistent, we means that the source always has enough data to transmit at the allocated rate.

A.4. There are no arrivals and departures of virtual circuits until the system reaches steady state.

A.5. The available bandwidth of the bottleneck link is constant until the system reaches steady state. Also, the buffer size at the bottleneck link is assumed infinite.

The assumptions **A.4-A.5** will be removed as necessary in Section 5 and in Section 6 to deal with dynamic environments.

Let $r_i(t)$, $i = 1, \cdots, n$, denote the rate allocation to virtual circuit i, which is computed by the switch at time t. Also, let $q(t)$, $\dot{q}(t)$ and μ respectively denote the queue length, its derivative at time t, and the available bandwidth at the bottleneck link. The rate-based flow control algorithm that we introduce in this paper is a switch algorithm of the following simple form:

$$r_i(t) = (\ -\frac{K_i}{n}(\ q(t) - q_T\ )\ )^+, \quad K_i > 0 \tag{1}$$

where K_i is the control gain, q_T is the queue length threshold for the flow control, and the symbol $(\ \cdot\ )^+$ denotes $max\{\ \cdot,\ 0\ \}$. We will refer to this algorithm as *First-Order Rate-Based Flow Control* (FRFC) since as we will see, the behavior of the closed-loop system with this form of algorithm is governed by a first-order differential equation. In contrast, most other existing algorithms[4–7,12] can be viewed as a second-order flow control since the rate is modulated via its derivative and thus the behavior of the closed-loop system is governed by a second-order differential equation. For instance, the algorithms[7,12] have the following form:

$$\dot{r}_i(t) = -a_i r_i(t) + b_i(\ q(t) - q_T\ ) \tag{2}$$

where a_i and b_i are two control constants. A feature of the algorithm (1) is the control gain K_i scaled by the number of VCs n. It will be shown later that such a scaling can lead to a uniform convergence of the bottleneck queue irrespective of the number of VCs. Another property of the algorithm (1) is that the closed-loop system has no stable equilibrium point when $q(t)$ is greater than or equal to q_T. In other words, as $q(t)$ grows and exceeds q_T, the rate allocation $r_i(t)$ becomes zero and thus with a delay the total arrival rate gets smaller than the available bandwidth at the bottleneck link. Thus, $q(t)$ cannot be asymptotically stabilized at a value greater than or equal to q_T. As we will see, $q(t)$ has only two equilibrium states; one at a positive value smaller than q_T and the other at zero depending on the network operational environment and the choice of the control gain parameters.

According to the above assumptions, the queueing process at the bottleneck link is given as

$$\dot{q}(t) = \begin{cases} \sum_{i=1}^{n} r_i(t-\tau_i) - \mu, & q(t) > 0 \\ (\ \sum_{i=1}^{n} r_i(t-\tau_i) - \mu\ )^+, & q(t) = 0. \end{cases} \tag{3}$$

In the next section we investigate the steady-state solutions and the asymptotic stability of these solutions when the control (1) is applied.

3. STEADY-STATE ANALYSIS AND STABILITY

We suppose that there exist equilibrium points for the closed-loop system and let q_∞ and $r_{i\infty}$ respectively denote the steady-state solution of $q(t)$ and $r_i(t)$. At equilibrium, we have $\lim_{t\to\infty} \dot{q}(t) = 0$ and thus from (3)

$$\sum_{i=1}^{n} r_{i\infty} - \mu = 0 \quad \text{if } q_\infty > 0, \tag{4}$$

$$(\ \sum_{i=1}^{n} r_{i_\infty} - \mu\ )^+ = 0 \quad \text{if } q_\infty = 0, \tag{5}$$

and from (1),

$$r_{i\infty} = (\ -\frac{K_i}{n}(q_\infty - q_T)\ )^+, \ \ \forall\ i. \tag{6}$$

First consider the case with $0 < q_\infty \le q_T$. From (4) and (6), we find that if $q_T > \frac{n\mu}{\sum_{i=1}^{n} K_i}$,

$$q_\infty = q_T - \frac{n\mu}{\sum_{i=1}^{n} K_i}; \quad r_{i\infty} = \frac{K_i}{\sum_{i=1}^{n} K_i}\mu, \ \ \forall\ i.$$

Next we consider the case with $q_\infty = 0 < q_T$. Similarly, from (5) and (6), we obtain that if $q_T \le \frac{n\mu}{\sum_{i=1}^n K_i}$,

$$q_\infty = 0; \quad r_{i\infty} = \frac{K_i}{n} q_T \le \frac{K_i}{\sum_{i=1}^n K_i}\mu, \ \ \forall\, i.$$

The following proposition states the result.

Proposition 3.1 *For FRFC, there are two steady-state solutions (equilibrium points): if* $q_T > \frac{n\mu}{\sum_{i=1}^n K_i}$,

$$q_\infty = q_T - \frac{n\mu}{\sum_{i=1}^n K_i}; \quad r_{i\infty} = \frac{K_i}{\sum_{i=1}^n K_i}\mu, \ \ \forall\, i, \tag{7}$$

otherwise,

$$q_\infty = 0; \quad r_{i\infty} = \frac{K_i}{n} q_T \le \frac{K_i}{\sum_{i=1}^n K_i}\mu, \ \ \forall\, i. \tag{8}$$

Note that q_∞ cannot be greater than or equal to the control threshold q_T. The network queue can be stabilized at either zero or a certain value smaller than q_T. For given $n\mu$, the choice of q_T and K_i, $i = 1, \cdots, n$, determines where for the system to converge. If q_T and K_i, $i = 1, \cdots, n$ are chosen such that $q_T > \frac{n\mu}{\sum_{i=1}^n K_i}$, the system has the equilibrium in (7) where the bottleneck link is fully utilized and any desired sharing of the bottleneck bandwidth can be accomplished through a proper selection of control gains. For instance, by taking identical gains, one can achieve MAX-MIN fair bandwidth sharing. On the other hand, if q_T and K_i, $i = 1, \cdots, n$ are chosen such that $q_T \le \frac{n\mu}{\sum_{i=1}^n K_i}$, the system has the equilibrium in (8) where the bandwidth sharing can be still fair but the available bandwidth cannot be fully utilized.

Now we investigate the asymptotic stability of the equilibrium point (7) where full link-utilization is achieved. Informally, asymptotic stability implies that all trajectories of the system, in this case the queue length and consequently the source rates as well, which start within some bound of the equilibrium point remain within another bound of the equilibrium point and further the trajectory asymptotically approaches the equilibrium point. The rigorous definition of asymptotic stability of equilibrium points can be found in the reference 10. In the following, we analyze only the local asymptotic stability and the global asymptotic stability is investigated via simulations only. For understanding the local stability, we can ignore the non-linearities introduced by the buffer floor, ceiling and the control, and the queueing process at the bottleneck link can be written as:

$$\dot{q}(t) = \sum_{i=1}^{n} r_i(t - \tau_i) - \mu \tag{9}$$

and

$$r_i(t) = -\frac{K_i}{n}(\ q(t) - q_T\ ) \tag{10}$$

respectively. Define

$$e(t) = q(t) - q_\infty = q(t) - q_T + \frac{n\mu}{\sum_{i=1}^n K_i}. \tag{11}$$

By combining (9), (10) and (11), we obtain the following closed-loop equation

$$\dot{e}(t) + \frac{1}{n}\sum_{i=1}^{n} K_i e(t - \tau_i) = 0 \tag{12}$$

which is a first-order retarded differential equation.[10,11] The characteristic equation of the closed-loop equation (12), denoted by $D(s)$, is

$$D(s) = s + \frac{1}{n}\sum_{i=1}^{n} K_i e^{-s\tau_i} = 0 \tag{13}$$

which is an exponential polynomial of s. For *asymptotic stability* of the closed-loop equation (12), all the roots of the characteristic equation (13) must have negative real parts.[10,11]

To find the necessary and sufficient condition for $D(s) = 0$ to have stable roots, one can appeal to Pontryagin's criterion[7,10] assuming discrete delays of rational ratios. For more general case with continuous delays or discrete delays of irrational ratios, to which our problem belongs, Stépán's criterion[11] provides a way to construct the necessary and sufficient condition. However, constructing such a condition in an explicit form is extremely complicated for the system with multiple connections of diverse round-trip delays.

Instead of finding the necessary and sufficient condition, we will derive a useful sufficient condition for the asymptotic stability of the equilibrium point (7). The following theorem in the reference 11, which we re-state below for convenience, provides the means to construct the necessary condition.

Theorem 3.1 *Consider the characteristic function $D(s)$ given by*

$$D(s) = \sum_{i=0}^{d} (-1)^i a_i(s) s^{d-i}. \tag{14}$$

Suppose that there exist the polynomials R^+, R^-, S^+ and S^- such that

$$R^-(\omega) \le R(\omega) \;=\; \text{Re } D(i\omega) \le R^+(\omega), \tag{15}$$

$$S^-(\omega) \le S(\omega) \;=\; \text{Im } D(i\omega) \le S^+(\omega), \tag{16}$$

for $\omega \in [0,\ \infty)$. Suppose that R^+ and R^- have the same number of real positive zeros. These zeros are denoted by $\rho_1^+ \ge \cdots \ge \rho_p^+ > 0$ and $\rho_1^- \ge \cdots \ge \rho_p^- > 0$ and they determine the intervals $I_{Rl} = [\ \min\{\rho_l^-, \rho_l^+\},\ \max\{\rho_l^-, \rho_l^+\}\ ]$, $(l = 1, \cdots, p)$. In exactly the same way, the zeros of S^+ and S^- define the intervals I_{Sj}, $(j = 1, \cdots, q)$. Furthermore, let all these intervals be disjoint and let us choose the representative real numbers $\rho_l^0 \in I_{Rl}$, $(l = 1, \cdots, p)$ and $\sigma_j^0 \in I_{Sj}$, $(j = 1, \cdots, q-1)$.

All the roots of the characteristic equation $D(s) = 0$ have negative real parts if

$$\begin{gathered} d = 2m, \\ S(\rho_l^0) \neq 0, \quad l = 1, \cdots, p, \\ \sum_{l=1}^{p} (-1)^l \text{sgn } S(\rho_l^0) = (-1)^m m; \end{gathered} \tag{17}$$

or

$$\begin{gathered} d = 2m+1, \\ R(\sigma_j^0) \neq 0, \quad j = 1, \cdots, q-1, \\ R(0) > 0, \\ \sum_{j=1}^{q-1} (-1)^j \text{sgn } R(\sigma_j^0) + \frac{1}{2}((-1)^q + (-1)^m) + (-1)^m m = 0 \end{gathered} \tag{18}$$

where m is integer and the symbol sgn *denotes sign function.*

The above theorem leads to the following result.

Proposition 3.2 *The closed-loop system (12) is asymptotically stable if $\frac{1}{n}\sum_{i=1}^{n} K_i\tau_i < 1$.*

Proof: From (13), we have $d = 1$ $(m = 0)$ and

$$R(\omega) = \frac{1}{n}\sum_{i=1}^{n} K_i \cos(\omega\tau_i),$$

$$S(\omega) = \omega - \frac{1}{n}\sum_{i=1}^{n} K_i \sin(\omega\tau_i). \tag{19}$$

Since $K_i > 0, \ \forall\, i$, the second condition in (18), $R(0) > 0$, is satisfied. On the other hand, one can estimate $S(\omega)$ in the following way:

$$S(\omega) > S^{-}(\omega) = (1 - \frac{1}{n}\sum_{i=1}^{n} K_i\tau_i)\omega, \quad \omega \in (0,\ \infty) \tag{20}$$

since $sin(\omega\tau_i) < \omega\tau_i$ for $\omega \in (0,\ \infty)$. If $\frac{1}{n}\sum_{i=1}^{n} K_i\tau_i < 1$, it is obvious that $S(\omega)$ has no real positive zeros since it is positive for $\omega \in (0,\ \infty)$. Thus, no σ_j^0s exist and hence the first and the second stability conditions in (18) are degenerated, independently from the polynomial estimation of $R(\omega)$. Theorem 3.1 implies the statement of the proposition. □

Since the above condition for stability is only a sufficient condition, the question naturally arises as to its tightness. Consider the case that $K_i = K$, $\forall\, i$. In this case, the condition implies that $K < {\tau_{avg}}^{-1}$ where τ_{avg} is the average round-trip delay of all the active VCs, i.e., $\tau_{avg} = \frac{\sum_{i=1}^{n}\tau_i}{n}$. It will be shown in the next section that the closed-loop system converges nearly exponentially and the gain K serves as the damping constant of the exponential decay. Since any reactive feedback-based flow control scheme in a wide-area network can only be expected to converge to the equilibrium point as fast as the average round-trip delay will permit, we hence conjecture that the condition for asymptotic stability is fairly tight in spite of being only a sufficient condition. The simulation studies in Section 6 will provide further evidence of the fast convergence even when the control parameters are set in accordance with the potentially restrictive sufficient condition. In addition, it is very difficult to find a sufficient and necessary condition, in an explicit form that is readily applicable, for the case of multiple connections with diverse round-trip delays[11]. In contrast, the condition in Proposition 3.2 is easy to use and requires only the estimate of average round-trip delay if identical gains are chosen.

The above stability analysis technique can also be applied to a higher-order flow control problem. Chong[12] has derived a sufficient condition for the asymptotic stability of a class of second-order flow control algorithms such as the one in (2) for the case of multiple VCs with diverse delays. Also, Elwalid[7] has applied the Pontryagin's criterion[10] to the second-order flow control problem.

4. TRANSIENT ANALYSIS

In this section, we consider the transient behavior of the ABR rate control algorithm whose behavior is governed by the first-order delayed differential equation in (12). or equivalently

$$\dot{q}(t) + \frac{1}{n}\sum_{i=1}^{n} K_i q(t-\tau_i) = \frac{1}{n}\sum_{i=1}^{n} K_i q_\infty, \tag{21}$$

subject to the initial conditions

$$q(t) = \begin{cases} q_0 & t = 0 \\ 0 & t < 0. \end{cases} \tag{22}$$

Consider the case that $K_i = K, \ \forall\, i$ and $\tau_i = 0, \ \forall\, i$. For this system, the solution of the above differential equation is:

$$q(t) = (q_0 - q_\infty)e^{-Kt} + q_\infty, \ \ t > 0. \tag{23}$$

Note that in this section, we use e to denote the exponential function which is not to be confused with the error function $e(t)$ defined in equation (11). We see above that the initial error in queue length dies exponentially fast with damping constant $-K$. The question of interest is whether for the retarded dynamical system this is still true.

For ease of illustration, we consider the case that $K_i = K$ and $\tau_i = \tau$, i.e., a homogeneous system with all sources at equal delay from the bottleneck queue. With these values for the gain and delay, we consider the solution $q(t)$ of the differential equation (21). The above equation can be solved using either the Laplace transform method or by forward deduction proceeding a time τ ahead in time each step and recognizing that the queue evolution equation does not change in this time period. The complete solution for $q(t)$ is then given as:

$$q(t) = q_0\sum_{j=0}^{\infty}(-K)^j\frac{(t-j\tau)^j}{j!}u(t-j\tau) - q_\infty\sum_{j=0}^{\infty}(-K)^{j+1}\frac{(t-j\tau)^{j+1}}{(j+1)!}u(t-j\tau), \tag{24}$$

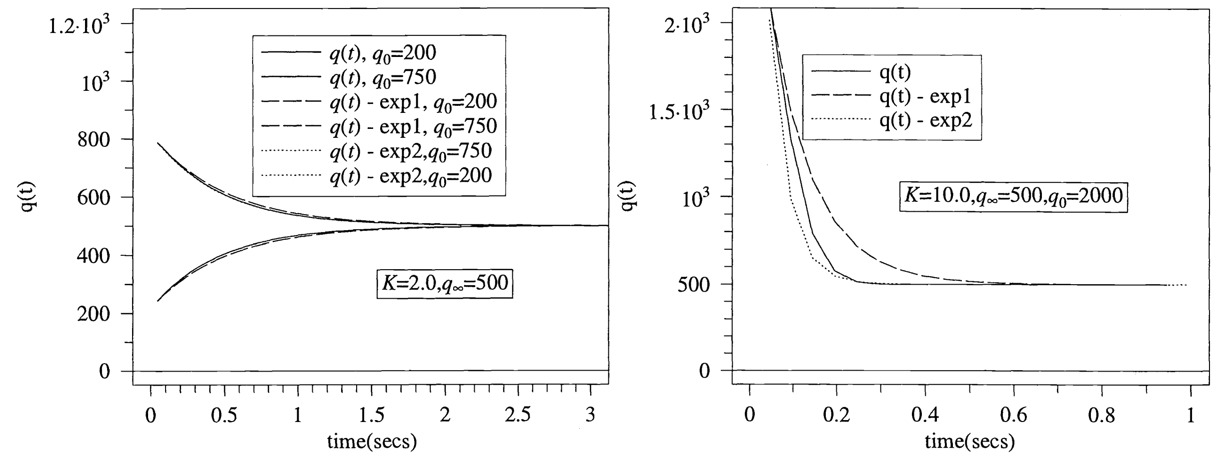

Figure 2: Transient behavior of the bottleneck queue: (a) low gain, (b) high gain.

where $u(\cdot)$ is the unit step function. The sums in the above term appear to be Taylor series expansions of the exponential function $e^{-K(t-\tau)}$ suggesting that the transient behavior of the retarded system also is exponential in nature. However, close examination shows the terms in the sum above do not exactly match the Taylor series expansions of the exponential function due to the term $t - j\tau$ and the unit step function. In fact, one can see that for $0 < t < \tau$, the behavior of $q(t)$ is:

$$q(t) = Kq_\infty t + q_0, \tag{25}$$

and hence the effect of the initial condition does not diminish at all in $(0, \tau)$. However, the close resemblance to the Taylor series expansion tempted us to consider an exponential approximation.

Our general exponential approximation is

$$q(t) \approx (q_0 - q_\infty)e^{\beta t} + q_\infty. \tag{26}$$

where β is derived by two different means. In the first approximation, which we label as exp1, we choose $\beta = -K$. In the second approximation, labeled as exp2, we obtain β by substituting the above approximation for $q(t)$ in equation (21) and solving the implicit equation

$$\beta + Ke^{-\beta\tau} = 0. \tag{27}$$

Note that no solution for β may exist in which case we select a value of β which minimizes the left-hand side of the above equation. Further, we found in our numerical studies that better approximations were obtained by matching $q(t)$ at time $t = \tau$ rather than at time $t = 0$ as warranted by the initial conditions above. This is due to the distinctly different behavior of $q(t)$ in $(0, \tau)$ as outlined previously. In this case, our general exponential approximation can be rewritten as:

$$q(t) \approx (q(\tau) - q_\infty)e^{\beta(t-\tau)} + q_\infty, \quad t > \tau. \tag{28}$$

In order to verify the accuracy of the above approximation , we carried out some numerical studies with different values of K and q_0 with $\tau = 40$ msec. Figures 2 a, b show the actual behavior of the queue and the approximation above for stable gains $K = 2.0$ and $K = 10.0$ respectively. It can be seen that the actual queue behavior is accurately tracked by both approximations for small K. In fact, with $K = 2.0$, the value of β obtained by solving equation (27) is $\beta = -2.18$ which is close to the value of β in exp1. For $K = 10.0$, we find that exp1 is considerably conservative compared to the actual behavior of $q(t)$ which is better approximated by exp2. In this case, we found no solution of β which satisfies equation (27); we choose a value of $\beta = -23.0$ which approximately minimizes the left-hand side of equation (27). This indicates that in this case $q(t)$ converges to its steady-state value almost twice as fast as the gain value $K = 10.0$.

While the exact transient performance can be easily computed as in equation (24), the exponential approximation above serves two purposes. First, it suggested to us that the effect of the initial condition dies out nearly exponentially in FRFC and second it helps us determine a thumb of rule for the transient response of the system. As a thumb of rule, we may choose $1/K$ as the time constant of the system, i.e., the effect of the initial condition diminishes by e^{-1} every $1/K$ time units. Finally, note that the transient behavior of $q(t)$ is independent of the number of active VCs, which is a highly desirable property in practice since it guarantees an identical speed of convergence no matter how many VCs are active. In contrast, it can be seen that if the gain is not scaled by the number of VCs in the control (1), the approximation exp1 of the transient behavior is given by

$$q(t) \approx (q_0 - q_\infty)e^{-nKt} + q_\infty \tag{29}$$

and nK should be smaller than $\frac{1}{\tau_{avg}}$ for asymptotic stability. What is undesirable in this type of control is that one needs to estimate the maximum number of possible VCs, say n_{max}, and the gain should be selected for this extreme case such that $K < \frac{1}{n_{max}\tau_{avg}}$. As a consequence, in nominal cases with n much smaller than n_{max}, the system would converge unnecessarily slowly due to the small value of the chosen gain.

The above analysis considered a homogeneous system. The exact transient behavior of the system for VCs with heterogeneous delays can also be written down in a form similar to equation (24). However, some more analytical work is needed to understand the qualitative transient behavior in this heterogeneous system as was done for the homogeneous system above.

5. CONTROL WITH DYNAMIC QUEUE THRESHOLD

So far we have studied the asymptotic stability and the steady-state solutions of the system in a static environment where the bottleneck bandwidth μ and the set of active VCs are assumed to be unchanged. In reality, however, these assumptions are no longer true. The available bandwidth at the bottleneck link is time-varying since it depends on the instantaneous aggregate traffic of higher-priority services such as CBR and real-time/non-real-time VBR in ATM. Also, the set of active VCs keeps changing due to the frequent arrival and departure of VCs. One of the major problem in such a dynamic environment is that the quantity $\frac{n\mu}{\sum_i^n K_i}$ changes and hence, as shown in Proposition 3.1, the equilibrium point (i.e., the steady-state solution) of the system varies. More specifically, if $\frac{n\mu}{\sum_i^n K_i}$ grows and exceeds q_T due to the changes in μ and the set of active VCs, the link would become under-utilized and the queue would converge to zero. On the other hand, if we choose q_T large enough to to avoid such a under-utilization of link bandwidth, q_∞ would grow and so the likelihood of cell loss would increase for given buffer budget.

What is desirable in such a dynamic environment is the capability to keep $q_T - \frac{n\mu}{\sum_i^n K_i}$, i.e., q_∞, constant and positive. To accomplish this, the switch can adaptively change either q_T or the control gains whenever the changes in μ and the set of active VCs are detected. Considering the large number of VCs in a high speed link, we choose the former option. The dynamic queue threshold (DQT) algorithm that we propose in this paper is to change the queue threshold in the following manner:

$$q_T(t) = \frac{|I(t)|\mu(t)}{\sum_{i\in I(t)} K_i} + \epsilon, \quad \epsilon > 0 \tag{30}$$

where $\mu(t)$ and $I(t)$ respectively denote time-varying bottleneck bandwidth and the set of active VCs with cardinality $|I(t)|$, and ϵ is a design parameter. For the case with $K_i = K$, $\forall\, i$, the above DQT is simplified to

$$q_T(t) = \frac{\mu(t)}{K} + \epsilon, \quad \epsilon > 0. \tag{31}$$

Consider the closed-loop system behavior with no buffer floor when the FRFC with DQT is applied. For simplicity we assume that only μ is time-varying while the number of VCs is fixed at n. Then, by combining (3), (1) and (30), we get the following closed-loop equation for the case with DQT

$$\dot{q}(t) + \frac{1}{n}\sum_{i=1}^{n} K_i q(t-\tau_i) = \frac{\epsilon}{n}\sum_{i=1}^{n} K_i - \mu(t) + \sum_{i=1}^{n} \frac{K_i}{\sum_{j=1}^{n} K_j}\mu(t-\tau_i). \tag{32}$$

In contrast, the closed-loop equation (12) for the case with static q_T can be rewritten as

$$\dot{q}(t) + \frac{1}{n}\sum_{i=1}^{n} K_i q(t-\tau_i) = \frac{q_T}{n}\sum_{i=1}^{n} K_i - \mu(t) \tag{33}$$

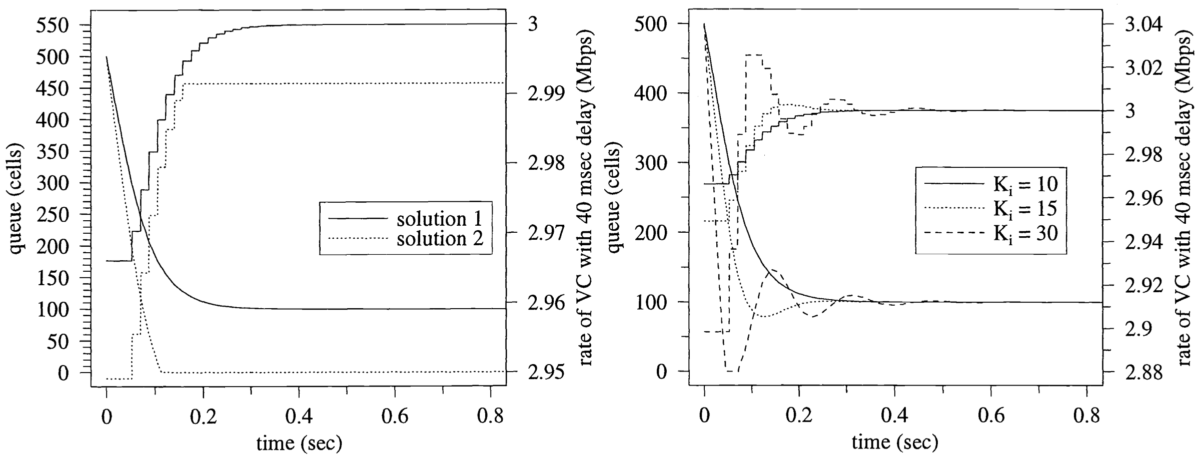

Figure 3: Control performance in static environment (μ = 150 Mbps, # of VCs = 50, $\tau_i \in$ [10, 40] msec, B = 5,000 cells): (a) two steady-state solutions with $K_i = 10.0$, $\forall\, i$, (b) effect of control gains K_i.

with time-varying $\mu(t)$. As we see in (32) and (33), the major difference between the DQT case and the static queue threshold case is the third term in the right-hand side of the DQT case. The role of this term is to nullify the effect of time-varying $\mu(t)$ with delays. In particular, if $\mu(t)$ varies slowly or is piecewise constant with reasonably long intervals, the term $-\mu(t) + \sum_{i=1}^{n} \frac{K_i}{\sum_{j=1}^{n} K_j}\mu(t-\tau_i)$ remains small in magnitude or as a superposition of impulses so that the effect of $\mu(t)$ becomes nearly nullified as the system approaches steady-state. In contrast, with a static threshold, the effect of $\mu(t)$ remains governing the dynamics of $q(t)$ as you see in (33). This difference will result in superior performances of the FRFC with DQT in dynamic environments, as will be shown in the next section through simulations.

The idea of dynamic queue threshold can be applied to queue-length-based rate control mechanisms of any arbitrary order. Chong[12] reports its application to a class of second-order flow control algorithms such as the one in (2).

6. PERFORMANCE

In this section, we simulate the network model to examine the performance of the proposed algorithm. First we consider a static scenario where the assumptions A.4 and A.5 hold. The bottleneck bandwidth μ and the buffer size B are respectively set to 150 Mbps and 5,000 cells, and there are 50 active VCs sourcing the traffic into the link. The round-trip delay τ_i of VCs is chosen uniformly in the range [10, 40] msec to represent long propagation delays. To take into account the discrete-time effect of control, the FRFC is applied in the sample-and-hold manner with intervals defined by the rate of VC. We choose this interval aggressively long as if RM (resource management) cells[1] are issued every 128 data cells by the ABR sources. Figure 3 a shows the two steady-state solutions, (7) and (8), with $K_i = 10.0$, $\forall\, i$, and $q_0 = 500$ cells. If we choose q_T at 35,477 cells, the queue $q(t)$ approaches 100 cells and the user rate $r_i(t)$ converges to the fair allocation (= 3 Mbps) as time goes. For the illustration, the rate trajectory of a VC with 40 msec round-trip delay is plotted in the figure. On the other hand, if we choose q_T at 35,277 cells, $q(t)$ converges to 0 and $r_i(t)$ approaches 2.9915 Mbps as computed in (8). Notice that it is not necessary for q_T be smaller than the buffer size B.

Figure 3 b shows the effect of control gains on the queue and user rates with $K_i = 10.0$, 15.0 and 30.0. In the above simulation scenario, the choice of $K_i = 10.0$, 15.0 and 30.0 satisfies the stability condition in Proposition 3.2 since $\frac{1}{\tau_{avg}} \approx 40$. While changing the gain, we kept ϵ $(= q_T - \frac{n\mu}{\sum_{i=1}^{n} K_i})$ positive and constant at 100 cells by changing q_T correspondingly. For the larger K_i, the system suffers from poor transient behavior such as overshoots at the risk of link under-utilization and cell loss, but still remains asymptotically stable. This example tells that the sufficient stability condition that we derive can serve as a practically good gain selection criterion.

In the remaining studies, we keep $K_i = 10.0$ unless otherwise specified. Note that this value of K_i is one of the

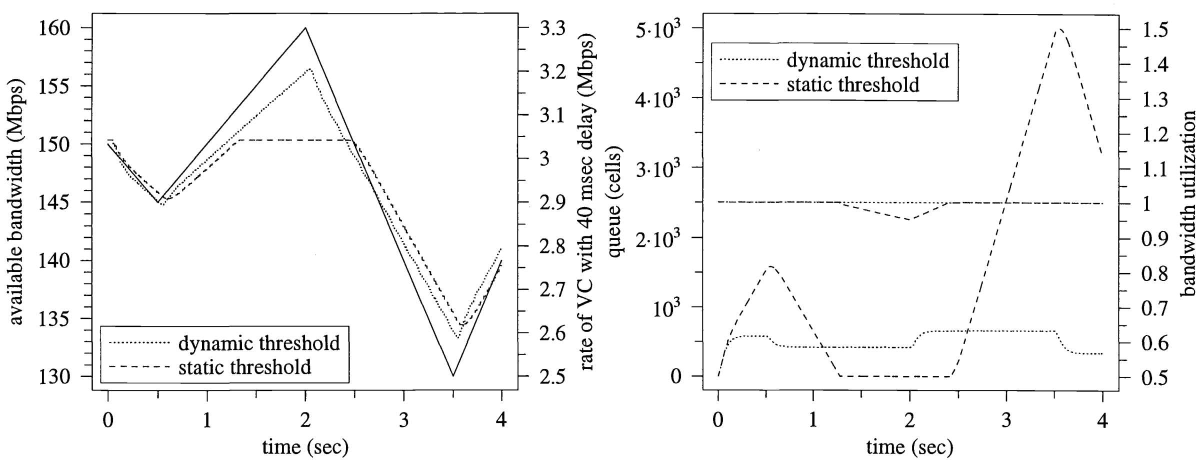

Figure 4: Control performance when $\mu(t)$ is continuously varying (# of VCs = 50, $\tau_i \in [10, 40]$ msec, B = 5,000 cells, K_i = 10.0, $\forall$ i): (a) trajectories of $\mu(t)$ (solid curve) and $r_i(t)$ of a VC with 40 msec round-trip delay, (b) trajectories of $q(t)$ and bandwidth utilization.

gain values examined in Section 4. We found there that for this gain value, the actual transient behavior (convergence rate) is faster than exponential for a homogeneous system. This provides additional confidence in our choice of gain.

Next we consider dynamic environments where μ and the set of active VCs are varying. For the FRFC with static threshold, we set q_T at 35,877 cells aiming at q_∞ = 500 cells with $\mu(0)$=150 Mbps. This design implies that if μ does not vary, $q(t)$ will converge to 500 cells. For the FRFC with DQT in (30), ϵ was fixed at 500 cells. First, we change $\mu(t)$ continuously in time with the derivatives of $\pm$ 10 and $\pm$ 20 Mbps/sec and apply the FRFC with/without DQT. The trajectory of $\mu(t)$ is plotted in Figure 4 a as a solid curve. Also, in Figure 4 a, the user rate $r_i(t)$ of a VC with longest round-trip delay (=40 msec) is compared for the two cases. With DQT $r_i(t)$ tracks well $\mu(t)$ with a time lag, whereas without DQT $r_i(t)$ suffers from loss of bandwidth as observed during the time interval [1.25, 2.5] sec. Figure 4 b explains why such a loss of bandwidth occurs without DQT. As explained in (33), with static threshold $q(t)$ essentially tracks the dynamics of $-\mu(t)$, consequently hitting both buffer floor and ceiling (see Figure 4 b). As also shown in Figure 4 b, the bandwidth utilization drops while $q(t)$ hits buffer floor, and hence the user suffers from the loss of bandwidth. On the other hand, if DQT is applied, $q(t)$ remains in the neighborhood of ϵ (= 500 cells), maintaining full utilization of bandwidth and no loss. This is because the time-varying dynamics of $\mu(t)$ does not directly affect the dynamics of $q(t)$. Rather, the difference between $\mu(t)$ and $\mu(t-\tau_i)$, $\forall$ i behaves as explained in (32).

Similarly, we compare the performance of FRFC with/without DQT when $\mu(t)$ is piecewise constant with 1 sec intervals. The trajectory of $\mu(t)$ is plotted in Figure 5 a as a solid curve. Exactly same observations can be made as in the previous scenario. With DQT, $q(t)$ remains in the neighborhood of ϵ, which is the design parameter, maintaining full utilization of bandwidth and no loss (see Figures 5 a, b). As explained in (32), it is observed in the trajectory of $q(t)$ in Figure 5 b that the jumps in $\mu(t)$ affect $q(t)$ as impulses so that the effect of jumps vanish after a certain transient period.

Finally, we consider a dynamic scenario where VCs arrive and depart. For simplicity we keep μ constant at 150 Mbps. For the FRFC with static threshold, we set q_T at 35,877 cells aiming at q_∞ = 500 cells with the given $I(0)$. This design implies that if $I(t)$ does not change, $q(t)$ will converge to 500 cells. For the FRFC with DQT in (30), ϵ was fixed at 500 cells. The trajectory of arrival/departure of VCs is plotted in Figure 6 a as a solid curve. Initially there are 50 VCs, 5 VCs simultaneously arrive at 1, 2 sec and 3 VCs depart at 3 sec. Also, in Figure 6 a, $r_i(t)$ of three representative VCs respectively arriving at 0 sec, 1 sec and 2 sec are shown only for the case with DQT. It is observed that the rates quickly converge to the fair share of the available bandwidth upon arrival and departure of VCs. The trajectories of $q(t)$ and the bandwidth utilization are found in Figure 6 b. Again, the FRFC with DQT outperforms the FRFC without DQT maintaining no loss, full utilization of available bandwidth and small queue. The spikes in $q(t)$ found in the case of the FRFC with DQT is due to the simultaneous arrivals of VCs. In practice,

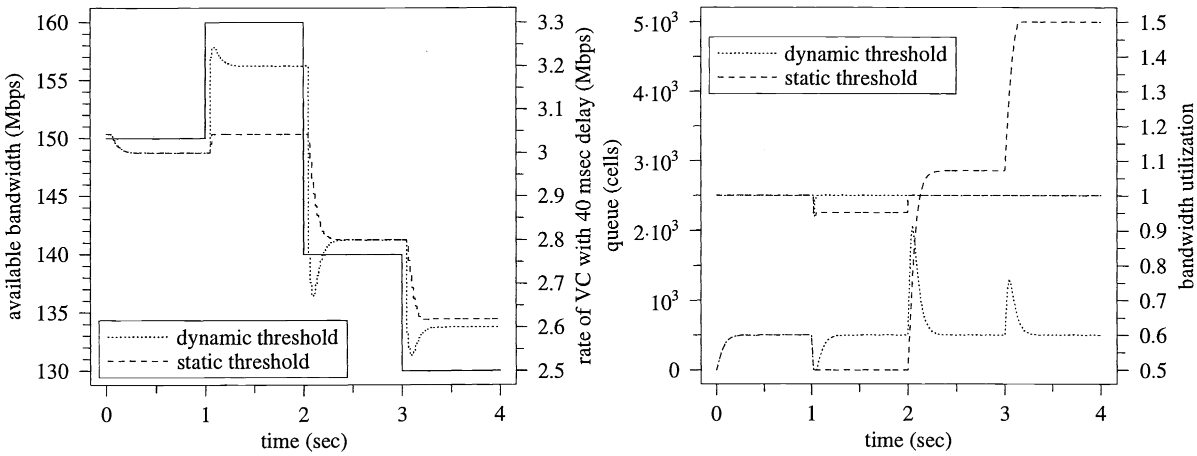

Figure 5: Control performance when $\mu(t)$ is piecewise constant (# of VCs = 50, $\tau_i \in [10,\ 40]$ msec, B = 5,000 cells, $K_i = 10.0, \forall\, i$): (a) trajectories of $\mu(t)$ (solid curve) and $r_i(t)$ of a VC with 40 msec round-trip delay, (b) trajectories of $q(t)$ and bandwidth utilization.

these spikes can be mitigated by applying a ceiling to $r_i(t)$s at the source point to restrict the rates, but at the cost of longer transient period.

7. CONCLUSION

In this paper we have introduced a new queue-length-based ABR flow control algorithm (FRFC). The asymptotic stability of the closed-loop system and the steady-state throughput, queue length and fairness have been analyzed for the general case of multiple connections with diverse round-trip delays. Further the stability condition and a complete characterization approximation and an useful practical approximation of the transient behavior have been derived. We have also presented a novel approach to dynamically adjust the queue threshold in the FRFC according to the changes of available bandwidth and arrivals and departures of connections. Through simulations, we were able to show that the FRFC with dynamic queue threshold effectively maintains high throughput, small cell loss and MAX-MIN fairness even in the dynamic environments. The simplicity and effectiveness suggests that the FRFC with DQT is a promising solution for ABR flow control in ATM networks. An extended analysis and a comprehensive cell-based simulation study for multi-hop configurations are under way and the results will be reported in a separate paper.

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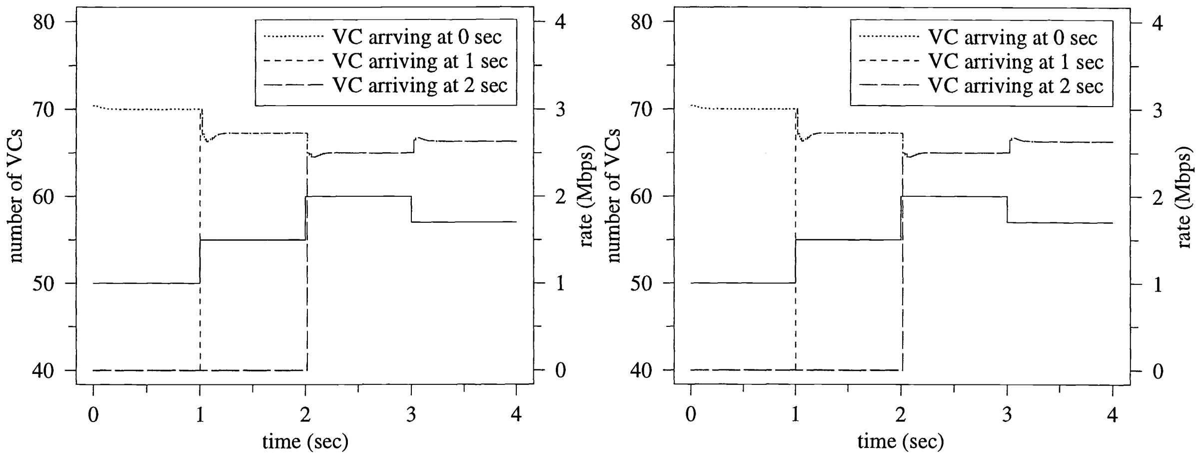

Figure 6: Control performance upon arrivals and departures of VCs (μ = 150 Mbps, $\tau_i \in [10, 40]$ msec, B = 5,000 cells, $K_i = 10.0, \forall\, i$): (a) trajectories of number of VCs (solid curve) and $r_i(t)$ of VCs arriving at 0, 1, 2 sec in the case of FRFC with DQT, (b) trajectories of $q(t)$ and bandwidth utilization.

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