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Utility Max-Min Flow Control Using Slope-Restricted Utility Functions†

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***Abstract*— We present a network architecture for the *distributed utility max-min* flow control of elastic and non-elastic flows where utility values of users (rather than data rates of users) are enforced to achieve max-min fairness. We provide a distributed link algorithm that does not use the information of users' utility functions. To show that the proposed algorithm can be stabilized *not locally but globally*, we found that the use of nonlinear control theory is inevitable. Even though we use a distributed flow control algorithm, it is shown that any kind of utility function can be used as long as the minimum slopes of the functions are greater than a certain positive value. We believe that the proposed algorithm is the first to achieve utility max-min fairness with guaranteed stability in a distributed manner.**

***Index Terms*— Utility max-min, nonlinear control theory, delayed systems, absolute stability, flow control**

I. Introduction

One of the most common understandings of fairness for a best-effort service network is *max-min fairness* as defined in [1]. There are several works [2]–[5] that provide distributed and stable max-min flow control algorithms that work in multiple bottleneck networks in spite of round-trip delays. Recently, Radunović and LeBoudec [6] considered not only max-min, but also min-max, fairness and observed that the existence of max-min fairness is actually a geometric property of the set of feasible allocations. Based on the relation between max-min fairness and leximin ordering, they completed a unified framework encompassing weighted and unweighted max-min fairness, and utility max-min fairness (to be explained) and provided a centralized algorithm that yields these fairness properties.

The rapid growth of multimedia applications has triggered a new fairness concept: *utility max-min fairness*. Originally, Cao and Zegura [7] introduced the concept of utility max-min fairness and motivated application-performance oriented flow control. They emphasized that applications have various kinds of utility function in general. For example, a voice over IP (VoIP) user corresponds to a step-like utility function because his satisfaction is at a maximum if the allowed rate is larger than the voice encoding rate and is at a minimum if the allowed rate is smaller than the encoding rate. The satisfaction of teleconference users with multi-layer streams, consisting of a base-layer stream and multiple enhancement-layer streams, would incrementally increase as additional layers were allowed. Therefore, to accommodate various types of application, it is necessary to relax the restriction on the shapes of utility functions as much as possible. To support various multimedia applications in multirate multicast networks, Rubenstein et al. [8] also employed utility max-min fairness. They showed that if multicast sessions are multirate, the utility max-min fair allocation satisfies desirable fairness properties that do not hold in a single-rate utility max-min fair allocation.

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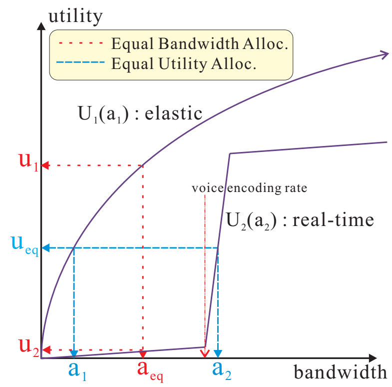

Fig. 1. Bandwidth max-min fairness versus utility max-min fairness.

In a single link case, utility max-min corresponds to the satisfaction (utilities) of each user in the network being equal. Let us consider a simple network in which a link of capacity μ is shared by two flows: an elastic flow with utility function $U_1(\cdot)$ and a real-time flow that transfers voice data with utility function $U_2(\cdot)$. As shown in Fig. 1, if the link capacity is shared equally (i.e., $a_{eq} = \frac{\mu}{2}$), the utility of the elastic flow, $u_1(= U_1(a_{eq}))$, becomes much larger than that of the real-time flow, $u_2(= U_2(a_{eq}))$, and the real-time flow is unsatisfactory because the allowed rate is smaller than the voice encoding rate. In contrast, if the link capacity is shared in a way that $U_1^{-1}(u_{eq}) + U_2^{-1}(u_{eq}) = \mu$, then both flows gain an identical utility (i.e., $U_1(a_1) = U_2(a_2) = u_{eq}$), and the real-time flow is satisfied with the allocation because the allowed rate is greater than the voice encoding rate. The former represents the bandwidth max-min fair allocation (equal bandwidth allocation in the single link case) whereas the latter represents the utility max-min fair allocation (equal utility allocation in the single link case).

There are several works [6]–[8] that present link algorithms to achieve the utility max-min fair bandwidth allocation, assuming that each link knows the utility functions of all the flows sharing the link. Note that the algorithms used in the cited studies are not distributed in the strict sense because they

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require global information, such as utility functions of users. Questions remain: (i) whether or not there exists a *distributed* link algorithm that does not require per-flow information, including utility function information, and (ii) whether or not such an algorithm converges in the presence of round-trip delays. As a solution to these questions, we provide a network architecture with a distributed flow control algorithm that achieves utility max-min fairness without using any kind of per-flow operations and provide stability results for the proposed flow control algorithm. In our proposed architecture, links do not need to know the utility functions of flows sharing the links.

Wydrowski et al. [9] proposed a somewhat similar architecture, although they did not mention utility max-min fairness. They considered a *linearized model* in which even gain values depend on the equilibrium point, which cannot be known in advance. Note that utility functions are naturally nonlinear and local stability results obtained through linearization techniques cannot guarantee global stability. It is very difficult to find a *region of attraction* [10] in such works. In contrast to this work, we consider a *nonlinear model* that does not exploit knowledge of the equilibrium point. To the best of our knowledge, this is the first work dealing with an analytical framework for the original problem and its stability.

The definition of utility max-min fairness is similar to that of bandwidth max-min fairness, except that utility values of users are max-min fair. Let us denote flow i's utility value and utility function by u_i and $U_i(\cdot)$, respectively. Two technical assumptions on $U_i(\cdot)$ for the analysis of the proposed network architecture are given as follows:

A.1. We assume that $U_i(\cdot)$ is a continuous and increasing function of user i's allocated data rate. By this assumption there always exists an inverse function of $U_i(\cdot)$, i.e., $U_i^{-1}(\cdot)$. It is quite natural that the values of utility functions increase as the allocated data rates increase.

A.2. We assume that $U_i(0) = 0$. It is also quite reasonable, since the utility function value of user i, i.e., the degree of user i's satisfaction, is zero when zero data rate is allocated.

Let us denote the set of all links, the set of all flows and the set of flows traversing link l by L, N and $N(l)$, respectively. Their cardinalities are denoted by $|L|$, $|N|$ and $|N(l)|$, respectively. Then, similar to the bandwidth max-min fairness [1], the utility max-min fairness can be defined as follows.

Definition 1: A rate vector $< a_1, ..., a_{|N|} >$ is said to be *feasible* if it satisfies $a_i \geq 0, \forall i \in N$ and $\sum_{i \in N(l)} a_i \leq \alpha_T^l \mu^l$, $\forall l \in L$.

Definition 2: A rate vector $< a_1, ..., a_{|N|} >$ is said to be *utility max-min fair* if it is feasible, and for each $i \in N$ and feasible rate vector $< \bar{a}_1, ..., \bar{a}_{|N|} >$ for which $U_i(a_i) < U_i(\bar{a}_i)$, there exists some i' with $U_i(a_i) \geq U_{i'}(a_{i'}) > U_{i'}(\bar{a}_{i'})$.

Here μ^l denotes the capacity of link l and α_T^l is a constant defining target link utilization of link l ($0 < \alpha_T^l \leq 1$). Let a vector $< u_1, ..., u_{|N|} >$ denote the utility vector corresponding to the rate vector $< a_1, ..., a_{|N|} >$ where $u_i = U_i(a_i)$, $\forall i \in N$. Then, Definition 2 can be restated more informally as follows: a rate vector $< a_1, ..., a_{|N|} >$ is said to be utility max-min fair if it is feasible and for each user $i \in N$, its utility u_i cannot be increased while maintaining feasibility without decreasing the utility $u_{i'}$ for some user i' for which $u_{i'} \leq u_i$.

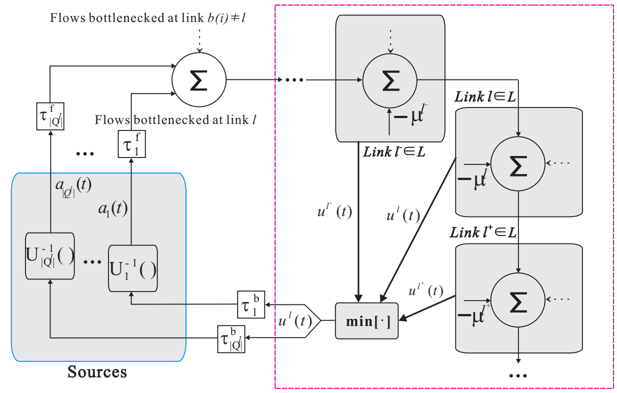

Fig. 2. The network architecture for utility max-min fairness.

Due to space limitation, readers are encouraged to refer to a Ph.D. thesis [11] for various utility functions and implementation issues. Proofs of theorems are also contained in [11].

II. Utility Max-Min Architecture

In this section, we propose a network architecture that achieves utility max-min fairness at equilibrium. The network architecture with multiple sources and links is depicted in Fig. 2. Let us consider a bottleneck link $l \in L$. Then, the dynamics of the buffer of the link can be written as

$$\dot{q}^l(t) = \begin{cases} \sum_{i \in N(l)} a_i(t - \tau_i^{l,f}) - \mu^l \quad , & q^l(t) > 0 \\ \left[\sum_{i \in N(l)} a_i(t - \tau_i^{l,f}) - \mu^l\right]^+, & q^l(t) = 0 \end{cases} \quad (1)$$

where $a_i(t)$ is the sending rate of source i, $\tau_i^{l,f}$ is the forward-path delay from source i to link l, μ^l is the link capacity of the link and the saturation function $[\cdot]^+ \triangleq \max[\cdot, 0]$ is such that $q^l(t)$ cannot be negative.

A sends packets according to the minimum utility value among the utility values assigned by the links along the path of its flow. Thus we assume the following *source algorithm*.

Source Algorithm: $$a_i(t) = U_i^{-1}\Big(\underbrace{\min_{l \in L(i)}[u^l(t - \tau_i^{l,b})]}_{u_i(t) \triangleq}\Big), \quad \text{(SA)}$$

where $L(i)$ is the set of links which flow i traverses, $u^l(t)$ is the utility value assigned by link l on the path of flow i, $\tau_i^{l,b}$ is the backward-path delay from link l to source i and $U_i(\cdot)$ is the user-specific utility function of user i. Because the min$[\cdot]$ operation is taken over a finite number of links, there should exist at least one link l such that $u^l = \min[\cdot]$. Therefore, each flow i has at least one bottleneck $l \in L(i)$.

There are two assumptions employed for the analysis of the network model.

B.1. We assume that the sources are *persistent* until the closed-loop system reaches steady state. We mean that the source always has enough data to transmit at the allocated rate.

B.2. $\tau_i^{l,f}$ and $\tau_i^{l,b}$ include propagation, queueing, transmission and processing delays. We denote the sum of two delays by τ_i and assume that this is constant.

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$$\textbf{Link Algorithm 1:} \quad u^l(t) = \left[-\frac{1}{|Q^l|} \left(g_P e_1^l(t) + g_I \int_0^t e_1^l(t)dt + g_D \dot{e}_1^l(t) \right) \right]^+ \tag{LA1}$$

$$\textbf{Link Algorithm 2:} \quad u^l(t) = \left[-\frac{1}{|Q^l|} \left(h_P e_2^l(t) + h_I \int_0^t e_2^l(t)dt + h_{I^2} \int_0^t \int_0^t e_2^l(t)dtdt \right) \right]^+ . \tag{LA2}$$

A. PID and PII² Link Controller Models

To control flows and to achieve utility max-min fairness, we use a PID link controller at each link. In the PID link controller model, there is a specified target queue length q_T^l to avoid underutilization of the link capacity. Because we have a nonzero target queue length q_T^l, the PID model implies that $\alpha_T^l = 1$ in Definition 1. Each link calculates the common feedback utility value $u^l(t)$ for all flows traversing the link according to the PID control mechanism.

Let us denote the set of flows bottlenecked at link l and its cardinality by Q^l and $|Q^l|$, respectively. The *link algorithm with PID controller* that uses the difference between $q^l(t)$ and q_T^l as input is given by Eq. (LA1) where $e_1^l(t) \triangleq q^l(t) - q_T^l$ is the error signal between control target and current output signal and, $g_P > 0$ and $g_I, g_D \geq 0$. It should be noted that we also can use a PII² controller as we did in [11], by defining $e_2^l(t) \triangleq \sum_{i \in N(l)} a_i(t - \tau_i^{l,f}) - \alpha_T^l \mu^l$ where $\alpha_T^l < 1$. The *link algorithm with PII² controller* is given by Eq. (LA2).

This model controls flows so that the queue length at steady state becomes zero at the cost of link underutilization. The main advantage of this model is that the feedback signal is not saturated at $q^l(t) = 0$ and it is shown through simulations in [11] that the PII² model results in faster convergence. In this paper, though we focus on the PID model to avoid repeating similar arguments for PII² model, readers should note that one can derive similar arguments regarding the PII² model with ease, as was done in [12].

Simple steady state analysis [11] reveals that the proposed network architecture possesses the utility max-min fairness property.

Theorem 1 (Utility Max-Min Fairness): The proposed network architecture described by Eqs. (1), (SA) and (LA1) (or (LA2)) achieves utility max-min fairness at steady state.

Proof: See Appendix in [11]. ■

III. Stability Analysis

Although we presented a multiple bottleneck network model in Section II-A, rigorous stability analysis of these kinds of models was shown to be very difficult in [3], due to the dynamics coupling among links that operate on a "first come first served" (FCFS) principle. In [3], [12], though such dynamics coupling exists in theory, the effect of coupling was shown to be negligible through various simulations. Recently, Wydrowski et al. [9] also showed that the dynamics coupling is of a very weak form. Thus, in this section, we drop the superscript l and the analysis is focused on a single bottleneck model. We conjecture that our analytical results can be extended to multiple bottleneck models without significant modification.

We provide a stability theorem when the saturation functions employed in Eqs. (1) and (LA1) are relaxed in a single bottleneck network. Although our main stability theorem assumes that flows experience the same forward-path and backward-path delays, we conjecture that our theorem will hold even if flows experience heterogeneous delays, when an upper bound of τ_is, i.e., $\bar{\tau} \geq \max_{i \in N}[\tau_i]$ is used.

A. Homogeneous-Delay Case

To analyze the homogeneous-delay case of the PID control model, let $\tau_i^f = \tau^f$, $\tau_i^b = \tau^b$, $\forall i \in Q$ and $\tau^f + \tau^b = \tau$. Then all flows experience the same forward-path and backward-path delay. By Eqs. (LA1) and (1), we obtain the following equation:

$$\ddot{u}(t-\tau^b) = -\frac{1}{|Q|} \left(\sum_{i \in Q} g_P \ddot{a}_i(t-\tau) + g_I \dot{a}_i(t-\tau) + g_D \dddot{a}_i(t-\tau) \right) .$$

Thus we can see that the following transfer function $G(s)$ defines the relationship between $-\sum_{i \in Q} a_i(t)$ and $u(t - \tau^b)$:

$$G(s) \triangleq \frac{g_P s + g_I + g_D s^2}{s^2} \exp(-\tau s). \tag{2}$$

By defining $\mathcal{U}(\cdot)$ as follows, we acquire the block diagram shown in Fig. 3(a), which is a feedback connection of $G(s)$ and an increasing and continuous nonlinearity $\mathcal{U}(\cdot)$.

$$\mathcal{U}^{-1}(u) \triangleq \frac{1}{|Q|} \sum_{i \in Q} U_i^{-1}(u). \tag{3}$$

Thus we can expect from Fig. 3(a) that an absolute stability theorem might be applicable to the proposed closed-loop system. Of various absolute stability criteria, we have found that Dewey and Jury's criterion [13] is suitable for our systems.

The first procedure when applying the criterion is to determine whether $G(s)$ is asymptotically stable because $G(s)$ itself without feedback is required to be asymptotically stable to apply the criterion. However, we can see that the transfer function $G(s)$ itself without feedback is not asymptotically stable because it has a double pole at $s = 0$. To overcome this problem, we use a loop transformation with a constant $h > 0$ and the resulting system is shown in Fig. 3(b). It should be noted that the modified system is identical to the original system. It is shown in [12] that the closed-loop system with feedback $\mathcal{U}^{-1}(u) = u$ (an identical function) is asymptotically stabilized when the gains $G_D \triangleq g_D$, $G_P \triangleq g_P\tau$ and $G_I \triangleq g_I\tau^2$ fall within a restricted area, shown in Fig. 4. Furthermore, it is also a proven fact that the closed-loop system is asymptotically stable for any $|\hat{Q}_w| \geq |Q_w|$. ($|\hat{Q}_w|$ and $|Q_w|$ are defined in [12]. In our system, $w_i = 1, \forall i \in Q$.)

To summarize, this result implies that the closed-loop system is asymptotically stable for $\mathcal{U}^{-1}(u) = hu$, $\forall h \in (0, 1]$ when we let $|\hat{Q}_w| = |Q_w|/h$. Hence we can see that $G(s)/(1 + hG(s))$ is asymptotically stable for any gain sets (G_D, G_P, G_I) falling within a restricted area shown in Fig. 4,

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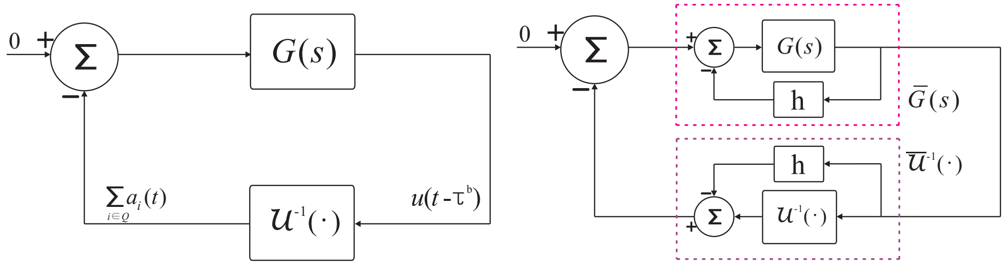

(a) original (b) after loop transformation

Fig. 3. Block diagrams of the proposed architecture

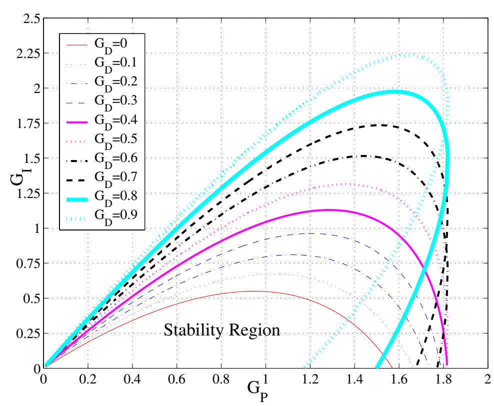

Fig. 4. Explicit stability region in terms of G_D, G_P and G_I when the feedback is an identical function.

and $h \in (0, 1]$. We are now ready to state the main result of this paper.

Theorem 2 (Homogeneous-Delay Case): The closed-loop system described by Eqs. (1), (SA) and (LA1) (or (LA2)) with the homogeneous-delay assumption $\tau_i^f = \tau^f$, $\tau_i^b = \tau^b$, $\forall i \in Q$ and $\tau^f + \tau^b = \tau$ is asymptotically stable for arbitrary utility functions with $0 < k \le dU_i/da < \infty$, $\forall a \in [0, \infty)$ and $\forall i \in Q$ if a gain set (G_D, G_P, G_I) falls within a restricted area shown in Fig. 4 and there exist a finite number η and a finite number $\kappa \ge 0$ such that the open-loop transfer function $G(j\omega)$ satisfies the following equation for arbitrarily small $h > 0$:

$$\mathrm{Re}\left[\left(1 + \frac{j\omega\eta}{1+\kappa\omega^2}\right)\frac{G(j\omega)}{1+hG(j\omega)}\right] + k > 0, \quad \forall \omega \ge 0. \quad (4)$$

Proof: For notational simplicity, we define two functions shown in Fig. 3(b) as follows.

$$\bar{G}(s) \triangleq G(s)/(1+hG(s)), \quad \bar{\mathcal{U}}^{-1}(u) \triangleq \mathcal{U}^{-1}(u) - hu.$$

By the assumption that (G_D, G_P, G_I) is contained in Fig. 4, we can see that $\bar{G}(s)$ is asymptotically stable for any $h \in (0, 1]$ from the arguments of Section 3.3 in [12]. Then we can apply the Dewey and Jury's criterion (Corollary 5 in [13]) to our nonlinear monotone feedback system because $\bar{G}(s)$ is asymptotically stable so that $g(t)$ and $\dot{g}(t)$ become elements of $L_1(0, \infty)$, i.e., the set of absolutely integrable functions and $\bar{\mathcal{U}}^{-1}(0) = 0$ by the assumption A.2. Although the differentiability of feedback nonlinearities was also assumed, this assumption is used only for the simplicity of their proof. If the feedback nonlinearities have left-hand and right-hand derivatives at all points, Dewey and Jury's criterion still holds.

If there exist a finite number η and a finite number $\kappa \ge 0$ such that the inequality (4) is satisfied for some small $h > 0$, then the closed-loop system is asymptotically stable with $\mathcal{U}^{-1}(u)$ satisfying the following equation by Dewey and Jury's criterion.

$$0 \le \frac{d}{du}\left(\mathcal{U}^{-1}(u) - hu\right) \le \frac{1}{k}.$$

If this is satisfied for arbitrarily small $h > 0$, we have the following condition for $\mathcal{U}^{-1}(u)$.

$$0 < \frac{d\mathcal{U}^{-1}(u)}{du} \le \frac{1}{k}. \quad (5)$$

When each of the utility functions, $U_i(a)$, satisfies $k \le dU_i/da < \infty$, then it also satisfies $0 < dU_i^{-1}/du \le 1/k$ and their sum becomes as follows, due to the finitude of $|Q|$.

$$0 < \frac{1}{|Q|}\sum_{i \in Q}\frac{dU_i^{-1}(u)}{du} \le \frac{1}{k} \iff 0 < \frac{d\mathcal{U}^{-1}(u)}{du} \le \frac{1}{k}.$$

Therefore, we can conclude that the closed-loop system is asymptotically stable if the minimum slope of the utility functions is restricted by k, i.e., $k \le dU_i/da < \infty$ $\forall$, $a \in [0, \infty)$. For PII^2 model, we can apply the same procedure because $G(s)$ of the PII^2 model is identical to that of the PID model. ■

Remark 1: One should note that this requirement is not stringent because the maximum slopes of the utility functions are not restricted, except for the condition that they should not be infinite. In other words, this restriction means that a user's satisfaction should increase with minimum slope of k for the stability of the whole network. A user can sufficiently emphasize that his satisfaction increases significantly at a certain data rate with relatively high slope at that data rate; because what matters is not the absolute shape of one's utility function, but its relative shape compared with those of others.

The most effective aspect of this theorem is that utility functions have only the minimum slope requirement and one user can use an arbitrarily-shaped nonlinear utility function that may differ from the other users' utility functions. We strongly believe that our requirement is one of the least

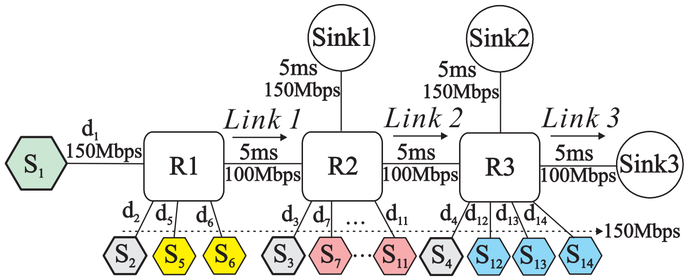

Fig. 5. Multiple bottleneck network used for Scenario 1

restrictive and most practical requirements in utility max-min network architecture.

B. Graphical Interpretation of Theorem 2

We know from [11] that the closed-loop system is asymptotically stable when $U_i(a) = a$ for all $i \in N$. Thus we can infer that Theorem 2 is meaningful only when there exists $k \leq 1$ satisfying Eq. (4). Even though it is difficult to find a k satisfying Eq. (4) for general cases, the inequality admits an intuitive graphical technique similar to the Nyquist stability criterion [14].

Corollary 1 (Explicit Range of k): For $G^3_{PID} \triangleq (G_D, G_P, G_I)$ = (0.242,0.868,0.261) and $G^2_{PID} \triangleq (G_D, G_P, G_I)$ = (0,0.482,0.091) that correspond to the PID and PI optimal gain sets, respectively, the minimum values of k are 0.480 and 0.338.

Proof: See Appendix in [11]. ■

Remark 2: Note that, for $\kappa = 0$, Eq. (4) reduces to the well-known Popov criterion, and the minimum slope constraint is dropped. One can easily verify through a graphical technique that the minimum slope constraint in our theorem is essential for getting a smaller k. Thus, instead of the Popov criterion, which has been regarded as one of the least conservative criteria when the nonlinear feedback $\phi(\cdot)$ is time invariant, we must use Dewey and Jury's criterion, which allows much smaller values of k thanks to the minimum slope constraint.

This corollary provides minimum values of k for two optimal gain sets. For the PID and PI controller models, respectively, with the gain set G^3_{PID} and G^2_{PID}, we can use any kinds of utility function that satisfy, respectively, $0.480 \leq dU_i/da < \infty$ and $0.338 \leq dU_i/da < \infty$. To introduce stability margins to the closed-loop system, it is recommended that the minimum slopes of utility functions be bounded by 1.

IV. Simulation Results

Using the four types of utility [11], i.e. premium utility, elastic utility, real-time utility, and stepwise utility, we provide several simulation results using ns-2 simulator [15] to demonstrate the merits of utility max-min flow control and the performance of our algorithms. In the following scenario, the largest round-trip propagation delays are set to 100ms. To avoid messy figures, we simulated our architecture with only two kinds of three-term link controllers, i.e., PID and PII^2 controllers. Simulation results for the PID and PII^2 link controller models are respectively denoted by G^3_{PID} and $G^3_{PII^2}$. For two-term link controllers, i.e., PI and II^2 controllers, we can obtain simulation results similar to those given in [12].

TABLE I
Flow Models Used for Scenario 1.

source	utility	d_i	begin(s) at	sink
S_1	premium	35ms	$-\infty$	Sink3
S_2	elastic	15ms	$-\infty$	Sink1
S_3	elastic	20ms	$-\infty$	Sink2
S_4	elastic	5ms	$-\infty$	Sink3
S_5, S_6	elastic	25, 30ms	10, 10.1s	Sink1
S_7, S_8, S_9, S_{10}, S_{11}	real-time	20, 40, 15, 40, 25ms	20, 20.1, 20.2, 20.3, 20.4s	Sink2
S_{12}, S_{13}, S_{14}	stepwise	30, 40, 10ms	40, 40.1s	Sink3

A. Scenario 1: Multiple Bottleneck Network With Heterogeneous Round-Trip Delays

To show that the proposed models work well in multiple bottleneck networks, we consider a network configuration in which there are three bottleneck links; see Fig. 5 where $\bar{\tau}$ = 120ms is used. The flow models used in this scenario is summarized in Table I. In Fig. 6, although there are queue overshoots at t=10s, 20s, 40s because several flows begin transmission simultaneously, such dramatic events (e.g., $S_7 \sim S_{11}$ begin transmission simultaneously.) do not occur frequently in real networks. In steady states, the sending rates of flows satisfy the feasibility condition in Definition 1 and utility max-min property in Definition 2, as shown in Fig. 7. Because S_1 traverses link 1, link 2 and link 3, $a_1(t)$ becomes nearly identical to the minimum of the feedback utilities at the three links, $\min[u^1(t), u^2(t), u^3(t)]$.

Four intervals are readily distinguishable; [$-\infty$, 10s], [10s, 20s], [20s, 40s] and [40s, ∞]. From $-\infty$ to t=10s, S_1 is bottlenecked at all three links. As new elastic flows, S_5 and S_6 destined for Sink1 begin transmission at t=10s, S_1 becomes bottlenecked only at link 1. Thus from t=10s to t=20s, flow S_3 and S_4 can send data at higher rates and S_2 can send data at a lower rate compared with the previous time interval, as shown in Fig. 7. As five real-time flows, $S_7 \sim S_{11}$ destined for Sink2 begin transmission at t=20s, S_1 is now bottlenecked at link 2. From t=20s to t=40s, S_2, S_5 and S_6 can send data at higher rates and S_3 can send data at a lower rate compared with the previous time interval. Similarly, when three stepwise flows destined for Sink3 begin transmission at t=40s, S_1 becomes bottlenecked at link 3 and flows are allocated bandwidth according to utility max-min fairness. Thus we can verify that our proposed algorithms work well in multiple bottleneck networks where the bottleneck link of a flow can change dynamically as the network situation changes.

V. Concluding Remarks

We have proposed a control-theoretic framework for application-performance oriented flow control. Our contribution is three-fold. First, we have found a distributed link algorithm that attains utility max-min bandwidth sharing while controlling link buffer occupancy to either zero or a target value. Moreover, the link algorithm does not require any per-flow information and processing, so it is scalable. Second, our algorithm is shown to be asymptotically stable in the presence of round-trip delays for arbitrary forms of utility function, as long as they are continuous and their slopes are larger than a certain positive constant. Third, our framework lends itself to a single unified flow control scheme that can simultaneously

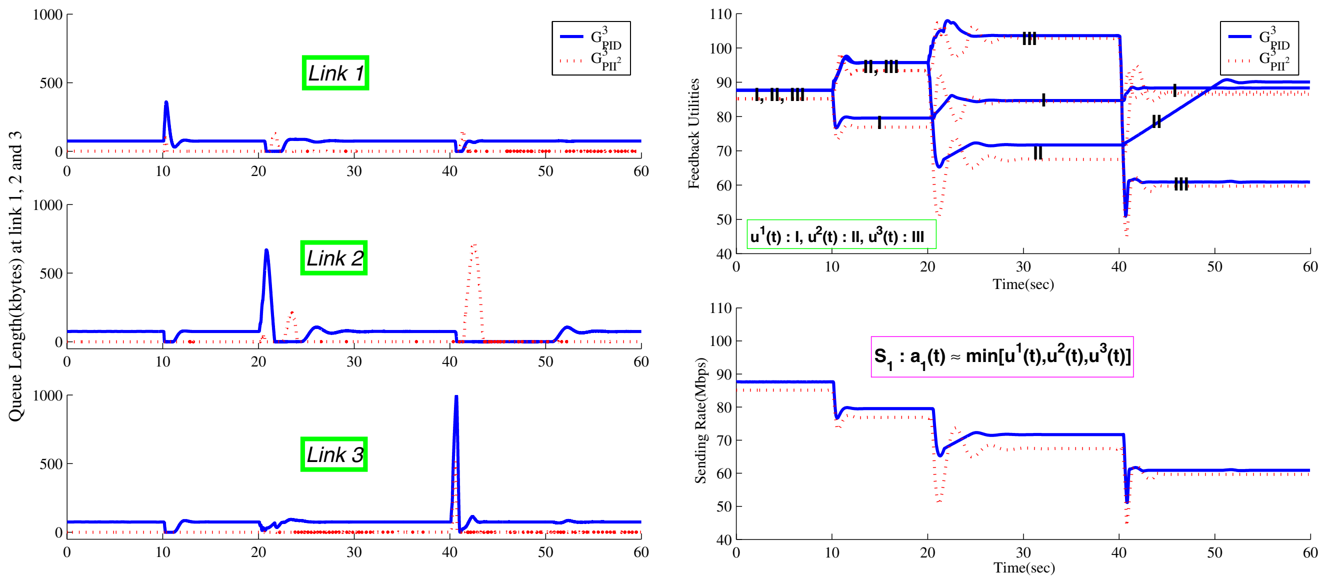

Fig. 6. Results of Scenario 1 - Queue length at links ($q^l(t)$), Feedback utilities at links ($u^l(t)$) and Source sending rate of S_1 ($a_1(t)$).

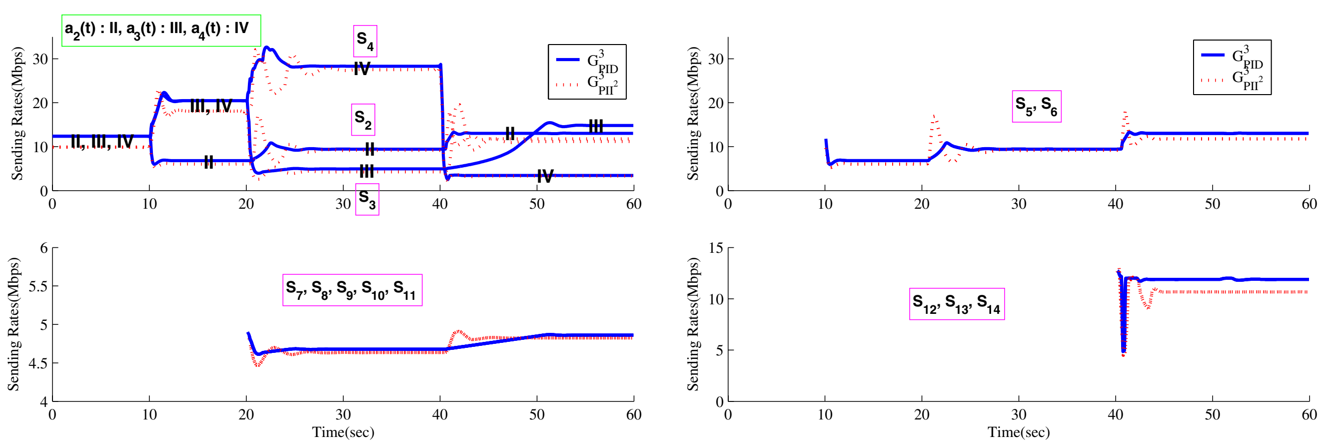

Fig. 7. Results of Scenario 1 - Source sending rates of $S_2 \sim S_{14}$ ($a_2(t) \sim a_{14}(t)$).

serve, not only elastic flows, but also non-elastic flows such as voice, video and layered video.

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