Infrastructure Support Increases the Capacity of Ad Hoc Wireless Networks*

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***Abstract*— In ad hoc wireless networks, one of the hottest research directions has been to increase the throughput capacity scaling with respect to the number of nodes n. Gupta and Kumar (2000) introduced a fixed random network model and showed that the throughput per source-destination pair is $\Theta(1/\sqrt{n \log n})$ when the network is coordinated by a centralized entity. Grossglauser and Tse (2001) introduced a mobile random network model and showed that a source-destination pair acquires a constant throughput of $\Theta(1)$ assuming that delays incurred by mobile nodes with low mobility are tolerable.**

In this paper, we consider the capacity of ad hoc wireless networks with infrastructure support. Although Grossglauser-Tse mobile network model enables $\Theta(1)$ per-node throughput scaling, the mobility assumption may be too unrealistic to be accepted in some practical situations. One of the key observations we acquired is that the infrastructure support plays the same role played by the mobility in the Grossglauser-Tse model. We show that nodes can utilize the randomly located infrastructure support instead of mobility when nodes are nearly static. In this case, we show that the per-node throughput of $\Theta(1)$ is still achievable when the number of access points grows linearly with respect to the number of nodes. Furthermore, we show that there is additional per-node throughput improvement of $\Theta(1)$ when nodes are mobile.

I. Introduction

Ad hoc wireless networks have been extensively studied since they can be used for future networks with various specific purposes. Because they can operate in an easily deployable, self-configurable, and highly flexible manner, it is clear that they will play an indispensable role in distributed wireless networks. A key concern in ad hoc wireless networks is the throughput scaling law that may depend on many aspects of networks such as power control, scheduling strategies, routing schemes, network topology, and physical characteristics. Since a thorough understanding of throughput scaling law can greatly affect the decision whether we should employ ad hoc wireless networks or not, recently many efforts have been devoted to improving the throughput performance of ad hoc wireless networks.

In their seminal paper [1], Gupta and Kumar showed that per-node throughput of $\Theta(1/\sqrt{n \log n})$ is attainable assuming that n number of mobile nodes are placed randomly. Roughly speaking§, $f(n) = \Theta(g(n))$ means that $f(n)$ and $g(n)$ are of the same rate of growth. This result is rather pessimistic since per-node throughput should decrease very fast as the number of mobile nodes increases. In the subsequent paper [2], Grossglauser and Tse showed that per-node throughput of $\Theta(1)$ is achievable when mobility is fully exploited. Although this result seems to be optimistic at the first glance, in some realistic situations, nodes are nearly static. Consequently, delays experienced by source-destination pairs will be intolerable [3] and the throughput per node becomes very small. However, we still think that it is possible to achieve the capacity of $\Theta(1)$, even if the nodes are static, which is a main result of this paper. In this paper, the mobility should not be an essential component but be an additional component that can potentially increase the throughput of ad hoc wireless networks.

To improve the throughput performance of ad hoc wireless networks without relying on mobility, we suggest to employ an infrastructure network where access points are randomly located in the network and they communicate to each other with relatively enough resources. Unlike cellular networks where base stations are located at the center of the cells and mobile nodes should directly communicate to base stations, in ad hoc wireless networks with infrastructure support, access points are randomly located without centralized coordination and nodes can communicate not only to access points but also to relay nodes and destination nodes. Since source nodes can send data to destination nodes utilizing the infrastructure network, ad hoc wireless networks can operate without relying on mobility.

A. Related Works

In the landmark paper of Gupta and Kumar [1], authors employed two models of interference: *protocol* model where a node can communicate with another node within a specified range and *physical* model where the impact of interfering signals on the signal-to-interference ratio (SIR) are considered in detail. They showed per-node throughput of $\Theta(1/\sqrt{n \log n})$ is attainable for *fixed random networks* and $\Theta(1/\sqrt{n})$ is also attainable if n number of nodes are placed and scheduled optimally. This result is gloomy because per-node throughput decreases approximately like $1/\sqrt{n}$ as the number of nodes increases.

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§Formally, $f(n) = \Theta(g(n))$ if there are constants $c_1 > 0$, $c_2 > 0$, n_0 such that $\forall n > n_0$ it is true that $c_1 g(n) < f(n) < c_2 g(n)$.

Grossglauser and Tse [2] introduced mobility into the ad hoc wireless networks and showed that per-node throughput of $\Theta(1)$ is achievable for *mobile random networks*. Although the result is very optimistic for dynamic situations where nodes move very fast, the delays experienced by nodes may become intolerable when nodes are nearly static. In a recent work [3], it is shown that the delay experienced by packets in Grossglauser-Tse model scales as $\Theta(\sqrt{n}/v(n))$ where $v(n)$ is the velocity of the nodes. Therefore, to allow bounded delays as the number of nodes goes to infinity, it is inevitable that the velocity of mobiles nodes scales as $\Theta(\sqrt{n})$, and such velocity scaling may not be possible in practical situations.

When we focus on the throughput capacity of ad hoc wireless networks with infrastructure support, there are two papers [4], [5]. Note that these works adopted the protocol model for their interference model. In [4], authors considered a network model composed of hexagonal cells and assumed that access points are located at the center of cells. They employed two different routing schemes that divides the nodes into two groups depending on whether they use the cellular network to reach their destinations or not. Each node belongs to one of two groups based on heuristic arguments that may not be optimal. Under the condition that the number of access points m grows faster than $\Theta(\sqrt{n/\log n})$, they showed that per-node throughput of $\Theta(m/n)$ is achievable. When $m = \psi n$ for a constant ψ, this result implies that per-node throughput of $\Theta(1)$ is achievable.

To the contrary, authors in [5] considered a network where access points are randomly located. Extending the work of Gupta and Kumar [1], they showed that per-node throughput of $\Theta(1/\log n)$ is achievable under the assumption that the number of nodes per access points is bounded above. Additionally, they showed that per-node throughput of $\Theta(1)$ is not achievable even if the network satisfies the connectivity in the weak sense: each nodes should be connected to at least one access point with high probability and this probability must be approaching to one as the number of nodes increases.

B. Our Contributions

Although two papers [4], [5] cited above and our paper have some overlaps, there are also significant differences as listed below that exhibit the contribution of our work:

1) First of all, in contrast to [4] and [5], we employ the physical model for our interference model that captures detailed characteristics of wireless networks such as path loss exponent α, interference amount I, thermal noise N_0 and the target SIR value β. Although the usage of physical model complicates analysis, we can consider detailed characteristics of ad hoc wireless networks and obtain more valuable information. Moreover, analytical results based on the physical model can be more appealing than those based on the protocol model.
2) Compared with [5], our work can be regarded as one that attains per-node throughput of $\Theta(1)$ by relaxing the concept of the connectivity in the weak sense introduced in [5]. If notable throughput improvement can be reached at the expense of the connectivity, some networks willingly choose to increase throughput capacity.
3) Compared with [4], our work also suggest that per-node throughput of $\Theta(1)$ is achievable purely depending on infrastructure network. But authors in [4] and [5] did not take mobility into consideration. We show that the per-node throughput improvement of $\Theta(1)$ can be additionally attainable when nodes are mobile.
4) In [4], access points are located at the center of hexagonal cells. In practical situations, it is not reasonable to assume that access points are located regularly. Furthermore, assuming random locations for access points can give us more accurate performance measure.

Additionally, we have observed that the role of infrastructure network is very similar to that of mobility in term of their contribution to throughput enhancement and we show their resemblance through insightful interpretation. We believe that this observation may necessitate the deployment of infrastructure network in situations where mobility cannot be assumed to be sufficiently dynamic.

The rest of this paper is organized as follows. In Section II, we describe our network model including the interference model and proposed scheduling policy. In Section III, we show that per-node throughputs of $\Theta(1)$ are achieved in pure ad hoc mode where only mobility is exploited, pure infra mode where only infrastructure is exploited, and hybrid mode where both of mobility and infrastructure are exploited. In Section IV, we obtain the exact forms of per-node throughputs by investigating the roles of mobility and infrastructure support in detail. Finally, we conclude this paper in Section V.

II. Network Model

We consider a random network with infrastructure support where mobility can be potentially exploited. There are two tiers where *infra tier* is an infrastructure network composed of access points and *ad hoc tier* is composed of nodes. All access points and nodes are randomly located in the disk of unit area (of radius $1/\sqrt{\pi}$). To make our model mathematically tractable, the bandwidth between any two access points is assumed to be relatively enough.

There are n nodes in the ad hoc tier. For each *slotted* time t, we randomly designate $n_S = \theta n$ of the nodes as source nodes and the remaining $n_R = (1-\theta)n$ nodes as destination nodes. The *source density* parameter θ falls within the open interval $(0, 1)$. Similar to [2], a destination node can be also a relay node. Since the designation changes at every time slot, the role of each node is varying. The number of access points n_A is assumed to be proportional to n, i.e., $n_A = \psi n$ where ψ is the *infrastructure density* parameter falling within $[0, 1]$. The ad hoc tier model used in this paper is not very different from the one employed in [2].

In the ad hoc tier, a source node or relay node sends data to other nodes at R bits/s through a single common channel. It is assumed that each access point has K uplink channels where each channel has the bandwidth of R bits/s. We assume the full orthogonality among K channels such that the transmissions occurring within one channel do not interfere with the other channels. There is the same amount of uplink traffic and downlink traffic because there is no traffic created in the infra tier. But we know that downlink traffic may not be evenly distributed to all access points but may be concentrated at a specified access point. For simplicity, we assume that downlink bandwidth is relatively abundant compared with uplink bandwidth.

A. Interference Model

For the interference model, we adopt a *physical* model where the main features of ad hoc wireless networks are specified with the signal power of a node and the interference signal from other nodes. At time slot t, let $P_i(t)$ be the transmission power of node i and $\gamma_{ij}(t)$ be the channel gain from node i to node (or access point) j, such that the received power at j is $P_i(t)\gamma_{ij}$. The transmission from i to j at rate R bits/s through channel c is successful if

$$\frac{P_i(t)\gamma_{ij}(t)}{N_0 + \frac{1}{L}\sum_{k\in S_c,\ k\neq i} P_k(t)\gamma_{kj}(t)} \geq \beta$$

where S_c is the set of senders transmitting through channel c, β is the signal-to-interference ratio (SIR) requirement for successful communication, N_0 is the background noise power, and L is the *processing gain* of the system. The channel gain is assumed to be

$$\gamma_{ij}(t) \stackrel{\text{def}}{=} \frac{1}{|X_i(t) - X_j(t)|^{\alpha}}$$

where α is the path loss exponent greater than 2 and $\{X_i(t)\}$ is the location of node i at time slot t.

B. Proposed Scheduling Policy

For the moment, we assume that the node location processes $\{X_i(t)\}$ are independent, stationary, and ergodic. Note that mobility is exploited in a very similar way to [2]. Each source node *distributes* its packets to the node nearest to the source node in every time slot. Because the location of each node is random in every time slot, a source node can spread the traffic stream between the source and the destination to a large number of intermediate *relay nodes* after sufficient number of time slots. Therefore, for a source-destination pair S-D, all the other $n-2$ nodes serve as relay nodes in steady state. It is also shown in [2] that it is actually sufficient to *relay only once*.

We extend the scheduling policy described in [2]. In each time slot, n nodes are divided into the set of source nodes $\mathcal{S}$ and the set of destination nodes $\mathcal{R}$. Destination nodes also play the role of potential receivers, i.e., relay nodes. Thus nodes in $\mathcal{R}$ will be also called *relay nodes*. Let us focus on a certain time slot t and its subsequent time slot $t+1$. There are two phases in scheduling policy π and a simplified version is depicted in Fig. 1.

Phase I (Source Nodes Transmit): Each source node S transmits packets to its nearest relay node R through the single common channel (arrow (1) in Fig. 1) and transmits packets to its nearest access point AP through K uplink channels (arrow (2) in Fig. 1) simultaneously. The source node S randomly selects one channel among K uplink channels. With very low probability of $\Theta(1/n)$, the nearest relay node R can be the

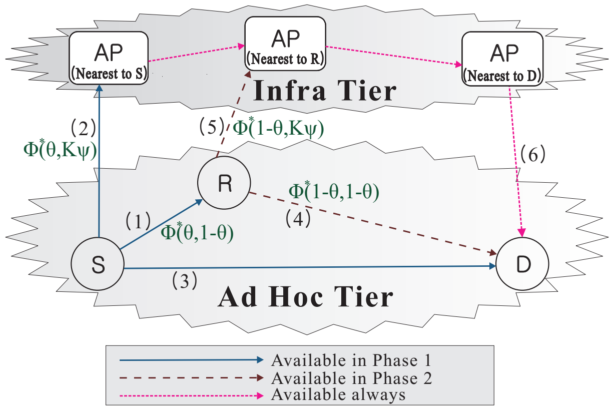


Fig. 1. The proposed scheduling policy.

true destination D of the source node (arrow (3) in Fig. 1). In this case, the source node S transmits packets directly to the true destination D without relaying.

Phase II (Relay Nodes Transmit): Each relay node R transmits packets to its nearest relay node only if the relay node R has packets whose true destination D is the nearest relay node (arrow (4) in Fig. 1). Concurrently, each relay node R transmits packets to its nearest access point AP through K uplink channels (arrow (5) in Fig. 1) in a random manner.

These two phases are interleaved: In the odd time-slots, Phase I is run. In the even time-slots, Phase II is run. Note that packets are *not* copied in any case. Once an access point receives packets, the packets are assumed to be delivered to the access point which is nearest to the true destination within a brief instant. Then the access point transmits the packets to the true destination (arrow (6) in Fig. 1).

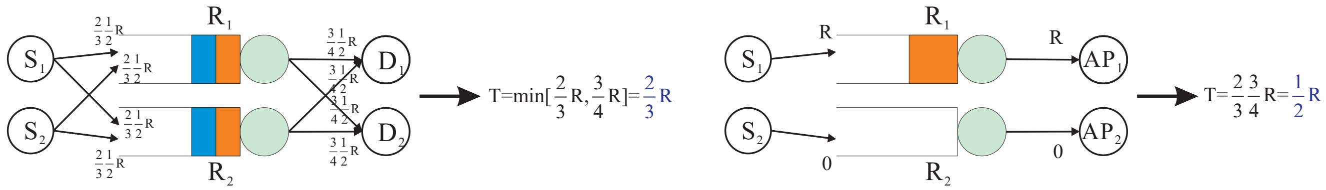
We can think of three kinds of mode: In pure ad hoc mode, arrows (1), (3) and (4) are used. In pure infra mode, arrows (1), (2), (5), (6) and intra infra-tier transmissions are used. In hybrid mode, all kinds of transmission are used. In pure ad hoc mode without infrastructure support, a packet delivered to a relay node should wait until the relay node rendezvous with the packet's true destination. To the contrary, in pure infra mode and hybrid mode where infrastructure is available, access points remove spatial obstacles between source nodes and their corresponding destination nodes and connect them. The infrastructure network not only *relays* packets but also *joins* distant nodes.

III. Capacity Improvement With Mobility and Infrastructure Support

In this section, we prove that ad hoc wireless networks operating in pure ad hoc mode, pure infra mode, and hybrid mode can achieve $\Theta(1)$ per-node throughput with mobility support. Theorem 1 shows that there are $\Theta(n)$ feasible S-D pairs in pure infra mode. (Its proof is in Appendix I.)

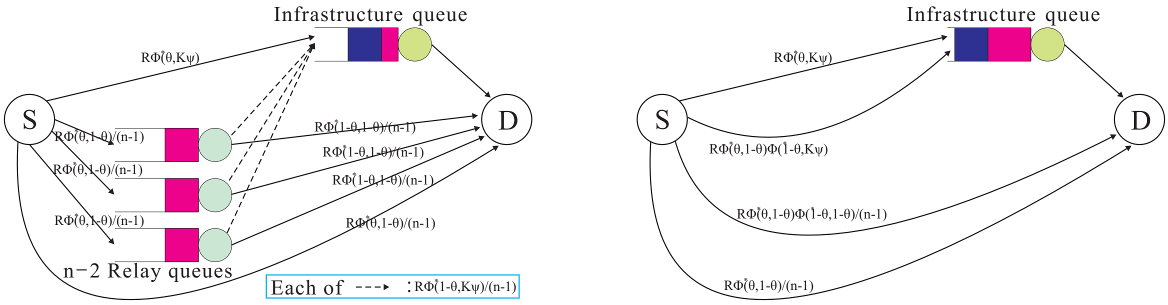
Theorem 1: For ad hoc wireless networks with scheduling policy π and the identical transmission power of $p \neq O(n^{-\alpha/2})^{\P,\dagger\dagger}$, the expected number $\mathrm{E}\{N_t\}$ of feasible source-access point pairs in Phase I is $\Theta(n)$, i.e.,

$$\lim_{n\to\infty} \frac{\mathrm{E}\{N_t\}}{n} = \Phi^*(\theta, K\psi) > 0 \qquad (1)$$



(a) When nodes are mobile. (b) When nodes are static.

Fig. 2. Simple examples illustrating per-node throughputs.



(a) When nodes are mobile. (b) When nodes are static.

Fig. 3. Two-phased scheduling policy viewed as queueing systems.

where $\Phi^*(\theta, K\psi)$ is defined in Eqs. (7), (8) and (9). Furthermore, the expected number $\mathrm{E}\{N_t\}$ of feasible relay-access point pairs in Phase II is $\Theta(n)$, i.e.,

$$\lim_{n\to\infty} \frac{\mathrm{E}\{N_t\}}{n} = \Phi^*(1-\theta, K\psi) > 0. \quad (2)$$

The above theorem says that the expected numbers of source-access point pairs and relay-access point pairs grow linearly with respect to the number of nodes n if the number of access points grows linearly with the number of nodes.

Let us turn to the ad hoc tier. Compared with transmissions from nodes to access points, transmissions from nodes to nodes can use only one common channel. Thus, in similar ways, the expected numbers of feasible source-relay pairs in Phase I, and relay-destination pairs in Phase II can be found. Four probabilities representing feasibilities of pairs are shown in Fig. 1.

From above arguments, we can see that the probability that a *specific* sender successfully transmits to *some* receiver is $\Theta(1)$ where sender and receiver can be either a node or an access point. As the node locations $\{X_i(t)\}$ are i.i.d., the probability that a *specific* sender successfully transmits to a *specific* receiver becomes $\Theta(1/n)$. In pure ad hoc mode, for a given S-D pair, there are one direct route and $n-2$ two-hop routes which go through one relay node R. For each two-hop route, we can consider the relay node R as a single server queue. Both the arrival rate and the service rate of this queue is the same and $\Theta(1/n)$. Summing over the throughputs of all $n-1$ routes including the direct route from S to D, it can be seen that the per-node throughput is $\Theta(1)$. In pure infra mode and hybrid mode, access points play the same role played by relay nodes in pure ad hoc mode. We have proved the following theorem, which is the main result of this paper.

Theorem 2: In pure ad hoc mode, pure infra mode, and hybrid mode, each node achieves throughput of $\Theta(1)$.

IV. The Roles of Mobility and Infrastructure Support

In this section, we investigate per-node throughputs in detail when nodes are mobile and static. This will disclose the roles played by mobility and infrastructure and show why infrastructure support is inevitable.

A. When Nodes Are Mobile

We first consider pure ad hoc mode with mobility assumption. To investigate the largest possible per-node throughput, we assume that a node rendezvous with all the other nodes with the same probability. To satisfy this assumption, the distribution function of $\{X_i(t)\}$ should be uniform. Then each source *evenly* distributes its packets to relay nodes. Let us assume that there are $2\bar{t}$ time-slots. Among all time-slots, a source transmits packets to relay nodes in $\Phi^*(\theta, 1-\theta)\bar{t}$ time-slots at a rate of R bits/s, and relay nodes transmit packets to the true destination node in $\Phi^*(1-\theta, 1-\theta)\bar{t}$ time-slots at a rate of R bits/s. Thus, as $\bar{t}$ goes to infinity, we can view this system as a queueing system where the arrival rate at relay nodes is $R\Phi^*(\theta, 1-\theta)$ and the departure rate from relay nodes is $R\Phi^*(1-\theta, 1-\theta)$. For example, let us consider two

[¶]Roughly speaking, $f(n) = O(g(n))$ means that $f(n)$ doesn't grows faster than $g(n)$. Thus, $f(n) \neq O(g(n))$ means that $f(n)$ grows faster than $g(n)$.

[††]For the case $p = \Theta(n^{-\alpha/2})$, Eqs. (1) and (2) are also satisfied. We do not deal with this case for brevity.

source nodes, their corresponding two destination nodes and two relay nodes as shown in Fig. 2(a). If $\Phi^*(\theta, 1-\theta) = \frac{2}{3}$ and $\Phi^*(1-\theta, 1-\theta) = \frac{3}{4}$, the arrival rate and departure rate of each relay node become $\frac{2}{3}R$ and $\frac{3}{4}R$. Thus the per-node throughput in this case becomes $\min[\frac{2}{3}R, \frac{3}{4}R] = \frac{2}{3}R$.

Thus the ad hoc wireless network can be viewed as a queuing system as shown in Fig. 3(a). Note that access points are grouped into one entity, i.e., infrastructure queue, and the downlink bandwidth from infrastructure queue to destination nodes are assumed to be $\Theta(1)$ and assumed to be sufficient to serve arriving packets. As n goes to infinity, the largest possible per-node throughput in pure ad hoc mode becomes

$$T^{\text{mobile}}_{\text{adhoc}} \stackrel{\text{def}}{=} \frac{R}{2} \cdot \min[\Phi^*(\theta, 1-\theta), \Phi^*(1-\theta, 1-\theta)]$$

where the factor $\frac{1}{2}$ is required since there are two phases and they are interleaved. Similarly, as n goes to infinity, the largest possible per-node throughputs in pure infra mode and hybrid mode become respectively

$$T^{\text{mobile}}_{\text{infra}} \stackrel{\text{def}}{=} \frac{R}{2} \cdot \{\Phi^*(\theta, K\psi) + \min[\Phi^*(\theta, 1-\theta), \Phi^*(1-\theta, K\psi)]\}$$

$$T^{\text{mobile}}_{\text{hybrid}} \stackrel{\text{def}}{=} \frac{R}{2} \cdot \{\Phi^*(\theta, K\psi) + \min[\Phi^*(\theta, 1-\theta), \Phi^*(1-\theta, 1-\theta) + \Phi^*(1-\theta, K\psi)]\}.$$

It can be easily observed that $T^{\text{mobile}}_{\text{hybrid}}$ is the largest among three throughputs. From above arguments, we can observe that the roles played by mobility and infrastructure are very similar when nodes are mobile.

Given a source-destination pair, both relay nodes and infrastructure network can be viewed as queueing systems serving their packets: Relay nodes can be viewed as $\Theta(n)$ multiple queues where the arrival rate and departure rate of each queue are $\Theta(1/n)$. The infrastructure network can be viewed as a queue where the arrival rate and departure rate of the queue are $\Theta(1)$. Thus, either mobility or infrastructure enables per-node throughput of $\Theta(1)$. The *fundamental reason* why we can achieve $\Theta(1)$ per-node throughput in pure ad hoc mode when nodes are mobile, is that packets originating from a source are distributed to multiple relay nodes since a source node and a relay node rendezvous with different nodes in each time slot.

B. When Nodes Are Static

When nodes are static, a source node transmits packets to a relay node with probability $\Phi^*(\theta, 1-\theta)$ or never transmits to relay nodes with probability $1 - \Phi^*(\theta, 1-\theta)$. Similarly, a relay node transmits packets to infrastructure network with probability $\Phi^*(1-\theta, K\psi)$ or never transmits any packet with probability $1-\Phi^*(1-\theta, K\psi)$. For example, let us consider two source nodes, two relay nodes and two access points as shown in Fig. 2(b). If $\Phi^*(\theta, 1-\theta) = \frac{2}{3}$ and $\Phi^*(1-\theta, K\psi) = \frac{3}{4}$, the probabilities that a source node transmits packets to a relay node and the relay node transmits packets to an access point successfully become $\frac{2}{3}$ and $\frac{3}{4}$. Thus the per-node throughput in this case becomes $\frac{2}{3} \cdot \frac{3}{4}R = \frac{1}{2}R$.

Thus, the expected throughput from a source node to infrastructure network through relay nodes becomes $R\Phi^*(\theta, 1-\theta)\Phi^*(1-\theta, K\psi)$ as shown in Fig. 3(b). Although a relay node having packets from a source node can also transmit packets to *some* destination node with probability $\Phi^*(1-\theta, 1-\theta)$, the probability that *some* destination node is the destination node of the source node is $\Theta(1/n)$. Thus, the expected throughput from a source node to its destination node through relay nodes becomes $R\Phi^*(\theta, 1-\theta)\Phi^*(1-\theta, 1-\theta)/(n-1)$ as shown in Fig. 3(b). As n goes to infinity, the per-node throughput in pure ad hoc mode becomes $T^{\text{static}}_{\text{adhoc}} = 0$. The per-node throughputs in pure infra mode and hybrid mode become

$$T^{\text{static}}_{\text{infra}} = T^{\text{static}}_{\text{hybrid}} = \frac{R}{2} \cdot \{\Phi^*(\theta, K\psi) + \Phi^*(\theta, 1-\theta)\Phi^*(1-\theta, K\psi)\}$$

Since $\min[x, y] > xy$ if $x, y < 1$, we can see that the following relation holds:

$$T^{\text{mobile}}_{\text{hybrid}} \geq T^{\text{mobile}}_{\text{infra}} > T^{\text{static}}_{\text{hybrid}} = T^{\text{static}}_{\text{infra}}.$$

The *fundamental reason* why we cannot achieve $\Theta(1)$ per-node throughput in pure ad hoc mode when nodes are static, is that packets originating from a source node are not distributed to multiple relay nodes but concentrated at a single relay node. It should be remarked that the usage of infrastructure network enables nodes to achieve per-node throughput of $\Theta(1)$ even if all nodes are static. Moreover, when nodes are mobile such that packets originating from source nodes are evenly distributed to relay nodes, additional $\Theta(1)$ per-node throughput is achieved.

V. Conclusion

In this paper, we have proposed ad hoc wireless networks with infrastructure support where nodes can exploit infrastructure network instead of mobility. Our contribution is three-fold. First, we have analyzed the throughputs of ad hoc wireless networks based on the physical model which captures essential characteristics of wireless networks. Thus we can say that our results are more convincing than others based on the protocol model. Second, we have shown that the per-node throughput in Grossglauser-Tse model approaches zero as the number of nodes goes to infinity if nodes are static since packets originating from a source node are concentrated at a single relay node. Third, we have shown that the usage of infrastructure network is essential in future ad hoc wireless networks since per-node throughput of $\Theta(1)$ is achievable even if nodes are static. We have observed that the roles played by mobility and infrastructure network are very similar in terms of their contribution to per-node throughput. Moreover, we can further improve per-node throughput in ad hoc wireless networks by exploiting mobility, and the per-node throughput improvement arising from mobility is also $\Theta(1)$.

Appendix I
Proof of Theorem 1

Proof: We consider a fixed time slot t. Let U_1, ..., U_{n_S} be the random positions of the sources in $\mathcal{S}$. Let V_1, ..., V_k be the positions of access points in $\mathcal{A}$ where $k = \psi n$. These random variables are i.i.d. uniformly distributed on the open disk of unit area. For each node $s \in \mathcal{S}$, let its intended access points $a(s) \in \mathcal{A}$ be the access point that is nearest to s among all access points in $\mathcal{A}$. Since there are K uplink channels and source node s chooses one channel randomly, without loss of generality, we assume that channel 1 is chosen by s.

We now analyze the probability of successful transmission for each source-access point pair. By symmetry, we can just focus on one such pair, say $(1,\ a(1))$. The event of successful transmission depends on the positions U_1, ..., U_{n_S} and V_1, ..., V_k. Let Q_i be the received power from source node i at access point $a(1)$, and

$$Q_i = p|U_i - V_{a(1)}|^{-\alpha}$$

The node $a(1)$ satisfies

$$a(1) = \arg\min_{j \in \mathcal{A}} p|U_1 - V_j|.$$

The total interference at access point $a(1)$ is given by $I = \sum_{i \neq 1} Q_i$. The SIR for the transmission from sender 1 at access point $a(1)$ is given by

$$\text{SIR} = \frac{Q_1}{N_0 + \frac{1}{L} I}$$

Step 1. Finding The Asymptotic Distribution of $\mathbf{Q_1}$

We now analyze the asymptotics of Q_1 and I as $n \to \infty$. Because p can be regarded as a constant,

$$Q_1 = \max_{j \in \mathcal{A}} Z_j$$

where $Z_j = p|U_1 - V_j|^{-\alpha}$. Following an analogous procedure used in [2], one can easily show that

$$\frac{1}{p(\pi\psi n)^{\alpha/2}} Q_1 \xrightarrow{\mathcal{D}} Q_\alpha^* \tag{3}$$

where Q_α^* has a cdf

$$F_{Q_\alpha^*}(x) = \begin{cases} \exp(-x^{-2/\alpha}), & x \geq 0 \\ 0, & x < 0. \end{cases}$$

Step 2. Rearrangement of I

Since each source randomly chooses a channel among K uplink channels between the source and its nearest access point, the total interference in a channel becomes a random sum of random variables Q_i. Since the probability that a sender chooses channel 1 is $1/K$, the total inference experienced by s becomes $I = \sum_{i \in S_1,\ i \neq 1} Q_i$ where S_1 denotes the set of sources that transmit data through channel 1. Then the cardinality of S_1 has the following binomial distribution.

$$\Pr\{|S_1| = m\} = \binom{n}{m} \left(\frac{1}{K}\right)^m \left(1 - \frac{1}{K}\right)^{n-m}$$

To rearrange I into a sum of a deterministic number of random variables, we now derive the generating function $\mathbf{E}\{s^I\}$ (See, e.g., [6]) of I. Using the fact that the generating function of a random sum is a compound function [7, pp. 287], $\mathbf{E}\{s^I\}$ becomes

$$\mathbf{E}\{s^I\} = \left(\frac{1}{K}\mathbf{E}\{s^{Q_i}\} + 1 - \frac{1}{K}\right)^n.$$

Therefore, we can regard I as a sum of $n_S - 1$ number of Q_i^+ which has a cdf

$$F_{Q_i^+}(x) = \begin{cases} 1 - \frac{1}{K} + \frac{1}{K} F_{Q_i}(x), & x > 0 \\ 0, & x \leq 0. \end{cases} \tag{4}$$

Step 3. Finding the Asymptotic Distribution of I

For the total interference $I = \sum_{i=2}^{n_S} Q_i^+$, the main difference is that we find an asymptotic distribution of summation of i.i.d. random variables. Following an analogous procedure used in [2], one can easily show that

$$\frac{1}{p\left[\pi\Gamma\left(1 - 2/\alpha\right)\left(\theta n - 1\right)/K\right]^{\alpha/2}} I \xrightarrow{\mathcal{D}} I_\alpha^* \tag{5}$$

where $\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$, and I_α^* is the stable distribution defined in [8].

Step 4. Asymptotic Independence of $\mathbf{Q_1}$ and I

Finally, we can easily show that I and Q_1 are asymptotically independent following an analogous procedure used in [2].

Combining this last fact with Eqs. (3) and (5), we get the result on the probability of successful transmission from source node 1 to access point $a(1)$.

$$\lim_{n\to\infty} \Pr\{\text{SIR} \geq \beta\} = \lim_{n\to\infty} \Pr\left\{\frac{Q_1}{N_0 + \frac{1}{L} I} \geq \beta\right\} \tag{6}$$
$$= \lim_{n\to\infty} \Pr\left\{\frac{p(\pi\psi n)^{\alpha/2} Q_\alpha^*}{N_0 + \frac{1}{L} p[\pi\Gamma(1-2/\alpha)(\theta n - 1)/K]^{\alpha/2} I_\alpha^*} \geq \beta\right\}$$

Furthermore, If $p \neq O(n^{-\alpha/2})$, that is, p grows faster than $n^{-\alpha/2}$ (e.g., p is a constant or $p = \Theta((\log n/n)^{\alpha/2})$, Eq. (6) becomes

$$\eta^*(\theta, K\psi) \stackrel{\text{def}}{=} \Pr\left\{\frac{Q_\alpha^*}{I_\alpha^*} \geq \beta^*(\theta, K\psi)\right\} \tag{7}$$

where

$$\beta^*(\theta, K\psi) \stackrel{\text{def}}{=} \frac{\beta}{L}\left[\frac{\theta}{K\psi}\Gamma\left(1 - \frac{2}{\alpha}\right)\right]^{\alpha/2} \tag{8}$$

which is also independent of n. Thus Eq. (1) is satisfied with

$$\Phi^*(\theta, K\psi) \stackrel{\text{def}}{=} \theta\eta^*(\theta, K\psi) \tag{9}$$

and the expected number of feasible source-access point pairs is $\Theta(n)$. In a similar way, we can verify Eq. (2) easily. ∎

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