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Discriminatory Congestion Pricing of Network Services: A Game Theoretic Approach Using Adverse Selection

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Abstract

The quality of network services deteriorates as network utilization increases beyond a certain point, i.e., congestion externalities. Many researchers have proposed congestion pricing models which internalize congestion externalities. However, most studies are based on a unit pricing which cannot reflect user's different congestion sensitivities. The purpose of this paper is to propose a discriminatory congestion pricing model using adverse selection. Our pricing mechanism provides a

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congestion-sensitive user with a high quality service for a higher price and a congestion-tolerant user with a low quality service for a lower price. Our model allows service providers to better control congestions while maximizing their profits.

Keywords: congestion pricing, congestion externalities, market segmentation, adverse selection

INTRODUCTION

The quality of network services such as Internet access and video on demand (VOD) services deteriorates as network utilization increases beyond a certain point. In such situations, additional (marginal) user causes delay or loss in the service (i.e., cost) for the rest of the service users. MacKie-Mason and Varian (1995a, 1995b) refer this state as a congestion externality. As the explosive usage growth in network services makes congestion externality problems very severe, prior research suggests that flat pricing should be replaced with usage pricing. However, usage pricing cannot control congestion since it does not reflect dynamically-changing network utilization. As a result, a user may get the quality he/she has paid for in low network utilization but he/she may not get the quality he/she has paid for in high network utilization. This is not an acceptable situation for service providers as well as for users (Rho and An 2003). Service providers may lose their money in the long run because dissatisfied users are not likely to return for their services. Therefore, service providers must control congestions.

One way to control congestions is to block new users from purchasing services when the number of current users is above certain thresholds (Bohn, et. al. 1993). Although this may be an effective and easy way to control congestion, it is not likely to maximize the profits of service providers (MacKie-Mason and Varian 1995a). Furthermore, it is not very user-friendly because some users may want to purchase services despite problems in the service quality. A better way to control congestion problems is to internalize congestion externalities by charging a higher unit price as congestion is getting worse (MacKie-Mason and Varian 1995a, 1995b). This congestion pricing mechanism is more user-friendly as users can either pay more for the better service quality or delay their purchase when congestion occurs.

Although most congestion pricing mechanisms internalize congestion externalities, they do not reflect users' different congestion sensitivities: some users are congestion-sensitive but others are congestion-tolerant. To solve this problem, we use adverse selection to design differentiated menus: a higher price for a high quality service to a congestion-sensitive user versus a lower price for a low quality service to a congestion-tolerant user. The purpose of this paper is to propose a discriminatory congestion pricing mechanism for a better congestion control as well as more profits for service providers. In the next section, we review the prior congestion pricing research. In the following section, we present a discriminatory congestion pricing mechanism. We then analytically compare our model with other pricing models and present a numerical example. Finally, we discuss implications of our results and directions for future research.

CONGESTION PRICING RESEARCH

Many researchers in information systems and computer science have studied the allocation problems to control congestion in network services. Since early 1990s, economics-based research has started to investigate network resource allocation problems (Lin, et. al. 2003). MacKie-Mason and Varian (1995a) show that a pricing mechanism can solve congestion externality problems. Since then, many papers have suggested that congestion pricing models should internalize congestion externalities by charging a congestion cost as well as a price for usage.

Varian (1995b) introduces a congestion pricing mechanism based on auction. Simiu and Srikant (1999) solve congestion externality problems by applying economic pricing mechanisms in network. Kelly (2000, 2001) proposes a congestion pricing mechanism to maximize social welfare. Basar and Srikant (2002) use a Stackelberg game to show a game-theoretic congestion pricing mechanism. Recently, Lin, et al. (2003) propose a pricing model to allocate computing resources such as network bandwidth and storage quantity so that total benefits of the service providers and the service consumers are optimized. Mathew, et al. (2004) suggest congestion pricing models based on congestion variations by showing the difference between Web service providers and application service providers (ASPs). All these pricing models design the same quality service for the same price to all users. Therefore, we can name such a pricing model as a unit congestion pricing model.

In reality, however, there are different types of users. Some are congestion-sensitive and the others are congestion-tolerant even though service providers do not know the information of users' private demand characteristics, i.e., which user is sensitive.

Gupta et. al. (2000) point out that most congestion pricing approaches do not consider such a realistic problem. They address the issue of estimating unknown delay costs based on users' choices in order to implement incentive-compatible network externality pricing. Based on the research of Gupta et al. (2000), Lin, et al. (2002) also study the issues in implementing congestion pricing mechanisms. Both studies implement more realistic congestion pricing mechanisms in network by considering asymmetric information situation but do not show how to segment a market for congestion-sensitive users and congestion-tolerant users. To design differentiated menus for both types of users, we suggest a game-theoretic discriminatory congestion pricing mechanism using adverse selection. We expect that a discriminatory congestion pricing helps service providers segment their markets and control congestion for higher profits and a more efficient resource allocation.

DISCRIMINATORY CONGESTION PRICING MECHANISM

We model our pricing mechanism using an adverse selection model assuming a monopolistic market with one service provider and two types of users: congestion-tolerant (type 1) users and congestion-sensitive (type 2) users. Table 1 summarizes the notations used in our model.

With notations in Table 1, we assume

1) q_i units of network resource are needed to provide the

Table 1. Notations

Symbol	Explanation
n_1 and n_2	Numbers of type 1 users and type 2 users
$N = n_1 + n_2$	Number of total users
q_1 and q_2	Quantities necessary to provide a low quality service and a high quality service, respectively
	Total quantity needed to provide services to users including new users
t_1 and t_2	Prices of a low quality service and a high quality service, respectively
K	Available system capacity at the time of new users' service requests
C	System capacity in the short run
θ_1 and θ_2	Congestion sensitivities (preferences) of type 1 users and type 2 users, respectively, where $\theta_1 \leq \theta_2$

service quality of q_i.

2) C and K are given.

3) θ_1 and θ_2 are users' private information.

Therefore, q_1 and q_2 measure the qualities that type 1 and type 2 users get, respectively. Since we consider a short-term period, network utilization can be represented as $Y= Q/C$ (i.e., the degree of congestion).

Since the capacity cost is a sunk cost in the short run, we consider only variable costs. According to MacKie-Mason and Varian (1995b), most costs of providing additional network services are more or less independent of the system utilization. As a result, we can model a service provider's profit function with only revenue from users since we can normalize variable cost to zero as in Varian (2000). Not knowing the congestion sensitivity of each user, a service provider can maximize its profit as stated in Salanie (1998).

$$\max_{t_1, t_2, q_1, q_2} n_1 t_1 + n_2 t_2 \tag{1}$$

Most congestion pricing models such as Basar and Srikant (2002), MacKie-Mason and Varian (1995a), and Simiu and Srikant (1999) internalize congestion externalities with user's utility function consisting of three terms: the utility of usage, the disutility of loss of quality due to congestion, and the price for quantities.

$$U_i = u_i\,(q_i;\ \theta_i) - v_i\,(q_i;\ q_{i-1},\ K) - t_i \tag{2}$$

Following their approach, we assume user 's utility function as follows:

$$U_I = \theta_I \log\ q_I - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_I \tag{3}$$

where qi ≥ 1 and K > $n_1 q_1 + n_2 q_2$

The first term is the utility of usage which is a strictly concave function satisfying Spence-Mirrlees condition ($u(q;\ \theta_2)$ is increasing in for all $\theta_2 > \theta_1$). The second term is the disutility from quality loss which is a strictly convex function as shown in Simiu and Srikant (1999). The last term is price. For simplicity, we normalize reservation price to zero. Then, a service provider lets each user participate in a transaction with the following two individual rationality (*IR*) constraints (i.e., participation constraints).

$$(IR_1)\ \ \theta_1 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1 \geq 0 \tag{4}$$

$$(IR_2)\ \ \theta_2 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_2 \geq 0$$

At the same time, a service provider has to design an incentive which makes each user to prefer the menu designed for him/her: type 1 users choose a low quality service for a lower price while type 2 users choose a high quality service for a higher price. A service provider can design such an incentive with the following two incentive compatibility (*IC*) constraints.

$$(IC_1)\ \ \theta_1 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1 \tag{5}$$

$$\geq \theta_1 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_2$$

$$(IC_2)\ \ \theta_2 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_2$$

$$\geq \theta_2 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1$$

Following Salanie (1998)'s approach, our model can be extended from two groups to n groups in sensitivity level with n incentive compatibility constraints. With (1), (4), and (5), a service provider designs differentiated menus for type 1 and type 2 users to maximize its profit while controlling congestion as follows:

$$\max_{t_1, t_2, q_1, q_2}\ \ n_1 t_1 + n_2 t_2$$

$$\text{subject to}\ \ (IR_1)\ \ \theta_1 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1 \geq 0$$

$$(IR_2)\ \ \theta_2 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_2 \geq 0$$

$$(IC_1)\ \ \theta_1 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1 \qquad (6)$$

$$\geq \theta_1 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_2$$

$$(IC_2)\ \ \theta_2 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) -$$

$$\geq \theta_2 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1 q_1 - n_2 q_2}\right) - t_1$$

where $K > n_1 q_1 + n_2 q_2$

In model (6), we can neglect (IC_1) and (IR_2) since (IR_1) andt (IC_2) are active and $q_2 \geq q_1$ (See Appendix for a proof). Then, we can formulate a discriminatory congestion pricing as shown in model (7).

Discriminatory congestion pricing model

$$\max_{t_1, t_2, q_1, q_2} \quad n_1 t_1 + n_2 t_2$$

$$\text{subject to} \;\; t_1 = \theta_1 \log \; q_1 - \frac{1}{N} \log\left(\frac{K}{K - n_1 q_1 - n_2 q_2} \right) \qquad (7)$$

$t_2 = t_1 + \theta_2 (\log q_2 - \log q_1)$
where $K > n_1 q_1 + n_2 q_2$

ANALYSIS

In this section, we analytically compare our pricing model with unit congestion pricing (unable to segment markets) and discriminatory usage pricing (unable to control congestion). We modify our discriminatory congestion pricing into the following models (8) and (9). (8) is a unit congestion pricing model which does not consider market segmentation whereas (9) is a discriminatory usage pricing model which does not consider congestion control.

Unit congestion pricing model

$$\max_{t, q} \; Nt$$

$$\text{subject to} \;\; t = \theta \;\; \log q - \log\left(\frac{K}{K - Nq} \right) \qquad (8)$$

where $K > Nq$

Discriminatory usage pricing model

$$\max_{t_1, t_2, q_1, q_2} \quad n_1 t_1 + n_2 t_2$$

subject to $t_1 = \theta_1 \log q_1$ (9)
where $K > n_1 q_1 + n_2 q_2$

The Necessity of Discriminatory Congestion Pricing Model

We first show why discriminatory congestion pricing is necessary using propositions 1 and 2. Proposition 1 compares total allocated quantities Q^*, Q^{**}, and Q^{***} of model (7), (8), and (9), showing the necessity of internalizing congestion externalities (See appendix for proofs).

Proposition 1: $Q^* \leq Q^{**} \leq Q^{***} = K$

Total allocated quantity of model (9) equals to the available system capacity. That is, a service provider allocates all available system resources under a discriminatory usage pricing model (9). When the network capacity is fully utilized, users experience severe congestions. In this case, users' disutility proportional to network utilization may be too significant for service providers to keep their users satisfied.

Total allocated quantities of congestion pricing models (7) and (8) are less than the available system capacity. Service providers can limit total allocated quantities to a certain threshold, preventing their users from experiencing severe congestions. Furthermore, allocated quantity of model (7) is less than or equal to that of model (8), which implies that the former is better at controlling congestion than the latter. In conclusion, market segmentation in addition to congestion pricing could further help service providers control congestion.

In proposition 2, we compare a discriminatory congestion pricing model (7) with a unit congestion pricing model (8) in terms of profits.

Proposition 2: $\Pi^* > \Pi^{**}$

Proposition 2 states that profit of model (7) is greater than that of model (8). Service providers employing model (7) can make more profits than that those employing model (8) even though resources allocated in model (7) are less than or equal to those in model (8). From propositions 1 and 2, we can infer that market segmentation help service providers make more profits as well as better control congestion.

Analysis of Discriminatory Congestion Pricing Model

Our model (7) designs differentiated menus for type 1 and 2 users as shown in proposition 3.

Proposition 3:

1) $q_1^* = \frac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)}K,\ \ q_2^* = \frac{\theta_2}{1+N\theta_1}$

2) $t_1^* = \theta_1 \log\ q_1^* - \frac{1}{N}\log\ (1+N\theta_1),\ \ t_2^* = t_1^* + \theta_2\ \ (\log\ q_2^*\ -\ \log\ q_1^*)$

3) $q_2^* > q_1^*,\ \ t_2^* > t_1^*$

where $q_1^* = t_1^* = 0$ if $\theta_2 < \frac{(1-n_1)N\theta_1 - n_1}{n_2}$ or θ_2

$$\geq \frac{N\theta_1}{n_2} - \frac{n_1}{n_1 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}$$

The equilibrium quantities and prices in model (7) are determined by the congestion preference and the number of each type user and total available system capacity. One of the interesting result is that model (7) allocates the available resources only to type 2 users when an additional usage makes the degree of congestion severe

$$(\theta_2 < \frac{(1-n_1)N\theta_1 - n_1}{n_2}\ \text{or}\ \theta_2 \geq \frac{N\theta_1}{n_2} - \frac{n_1}{n_1 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}.$$

What this result implies is that our method allows service providers to block a certain type of users from accessing network services when the network congestion is severe.

In proposition 4, we compare two differentiated menus in terms of unit prices.

Proposition 4: 1) $p_2^* < p_1^* = 0$ if $\theta_2(\log\ q_2^* - \log\ q_1^*) < (q_2^* - q_1^*)p_1^*$

2) $p_2^* < p_1^* = 0$ if $\theta_2(\log\ q_2^* - \log\ q_1^*) > (q_2^* - q_1^*)p_1^*$

$$\text{where } p_1^* = \frac{t_1^*}{q_1^*},\ \ p_2^* = \frac{t_2^*}{q_2^*}.$$

Proposition 4 states that the unit price of a high quality service may be lower or higher than that of a low quality service depending on their congestion preferences and network utilization. If a type 2 user's utility increment by switching menu from a type 1's to his/her own is less than the payment change (i.e. quantity change times the price for the type 1), he/she does not have any incentive to choose his/her own menu. In this case, a service provider has to discount a unit price to induce the type 2 user to select his/her own option (i.e., quantity discount). As a result, type 2 users get the quantity discount by revealing their private information when the network utilization level is low.

If a type 2 user's utility increment is more than the payment change, he/she has incentive to choose his/her own menu. In this case, a service provider can charge a premium to the type 2 user (i.e., quantity premium). Type 2 users must pay the quantity premium for causing more congestion externalities when the network utilization level is high.

Our adverse selection model is different from a traditional adverse selection model (model (9) which does not consider congestion externalities) in that type 2 users may experience a penalty (quantity premium) as well as a benefit (quantity discount) depending on their congestion preferences and network utilization. Type 2 users in most adverse selection studies get the quantity discount for revealing private information. However, type 2 users in this study can pay the quantity premium for causing more congestion externalities in high network utilization.

NUMERICAL ILLUSTRATIONS

In this section, we show the difference between models (7) and (8) using the following numerical example: C = 100, $n_1 = n_1 = 1$, $\theta_2 = 1.5\theta_1 = 1.5$. Table 2 shows the equilibrium quantities, prices, total quantities, and profits from models (7) and (8) depending on the level of network utilization.

Table 2. The Equilibrium Quantities, Prices, Total Quantities, and Profits

Utilization	Discriminatory Congestion Pricing Model (7)						Unit Congestion Pricing Model (8)			
	q_1^*	q_2^*	t_1^*	t_2^*	Q^*	Π^*	q^{**}	t^{**}	Q^{**}	P^{**}
0%	16.67	50.00	0.98	1.70	66.67	2.68	33.33	1.28	66.67	2.57
10%	15.00	45.00	0.94	1.65	60.00	2.59	30.00	1.24	60.00	2.48
20%	13.33	40.00	0.89	1.60	53.33	2.49	26.67	1.19	53.33	2.37
30%	11.67	35.00	0.83	1.54	46.67	2.37	23.33	1.13	46.67	2.26
40%	10.00	30.00	0.76	1.48	40.00	2.24	20.00	1.06	40.00	2.12
50%	8.33	25.00	0.68	1.40	33.33	2.08	16.67	0.98	33.33	1.97
60%	6.67	20.00	0.59	1.30	26.67	1.89	13.33	0.89	26.67	1.77
70%	5.00	15.00	0.46	1.18	20.00	1.64	10.00	0.76	20.00	1.52
80%	3.33	10.00	0.28	1.00	13.33	1.28	6.67	0.59	13.33	1.17
90%	–	5.00	–	0.72	5.00	0.72	3.33	0.28	6.67	0.57
100%	–	–	–	–	–	–	–	–	–	–

In Table 2, network utilization is measured by the ratio of the total quantity used and the system capacity, $(C - K)/C$. For example, 80% of network utilization implies the available system capacity at the time of new user's service request, K, is 20. The notation '-' represents a situation when a service provider does not provide the service.

In this example, model (7) provides each user with a differentiated menu until the network utilization is 80%. However, it blocks type 1 users from getting services when the network utilization is over 80%. On the other hand, model (8) provides one menu for both type of users until the utilization is 90%. The total quantities allocated in model (7) are less than or equal to those in model (8). However, model (7) provides a service provider with higher profits than model (8) does.

Figure 1 shows the unit prices of models (7) and (8) depending on the network utilization. In model (7), the unit prices generally increase as the network gets congested. The unit price of type 2 users is less than that of type 1 users until Point B in Figure 1 shown in Proposition 3. What this implies is that until a certain threshold of congestion, a service provider should provide quantity discount to type 2 users to induce the usage. When the network gets congested, the unit price of type 2 users increases

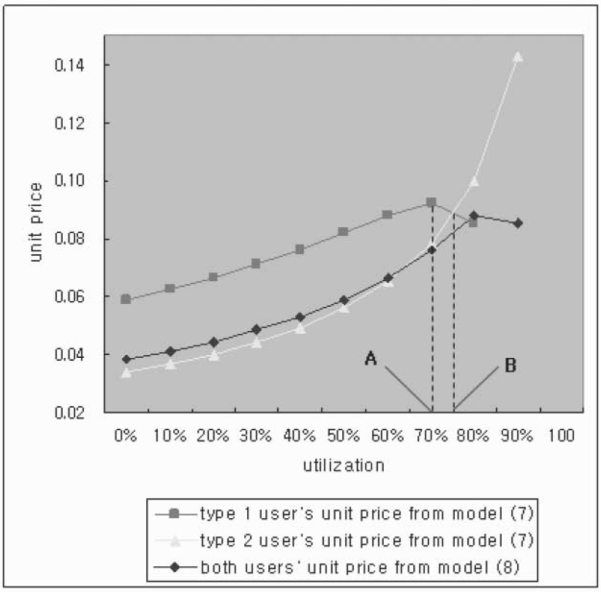

Figure 1. Unit Prices

rapidly. This implies that a service provider should charge premium to type 2 users to control congestions while maximizing profits. Unlike the unit price of type 2 users, the unit price of type 1 users decreases when the network is congested (over Point A). Point A is a point when type 1 users are no longer willing to pay more for each unit because the disutility due to congestion decreases the values of the service.

CONCLUSIONS

This paper has proposed a discriminatory congestion pricing mechanism for network services using adverse selection in game theory. Equilibrium quantities and prices in our model are determined by total available system capacity, the congestion preferences, and the proportion of each type users. Therefore, service providers can offer differentiated menus for users with different congestion sensitivities depending on dynamically-changing network utilization.

There are several implications for managers of network services. First, varying prices of network services depending on network utilization is an effective way of controlling congestions. Services subject to congestions such as VOD and network storage services require efficient and effective congestion control methods. However, most service providers solve congestion problems by increasing capacity, which is an expensive option. Our model provides an inexpensive alternative to increasing capacity.

Second, segmenting markets can not only maximize profits but also better control congestions. In most cases, market segmentation is used to increase profits. However, our model shows that market segmentation in congestion pricing allows better congestion control. Our model allocates less resources by blocking low quality service users from getting services and charging more for high quality service users.

Third, quantity premium as well as quantity discount for high quality services are necessary depending on network utilization. High quality service users in most cases get quantity discount for revealing their private information. However, high quality service users (i.e., type 2 users) in this study may pay the quantity premium for causing more congestion externalities in high network utilization. .

Our research has a few limitations, each of which is a direction for future research. First, we will extensively compare our model with other models using various sets of data. We will investigate the effects of various factors such as utilization, user sensitivity, proportion of each type users, etc. Second, we will extend our model in several directions. We will consider n types of users, because in real world, consumers' sensitivity levels would be much more heterogeneous than two levels assumed in this paper. We can also consider other factors such as administration cost and initial cost which are assumed to be zero in this paper. Finally, we will address the implementation issues of congestion pricing models such as ours. They include how to estimate the cognitive costs of users and overcome users' resistance, etc.

Appendix

Simplifying model (6) into model (7)

① Since $\theta_2 \geq \theta_1$ and $q_1 \geq 0$, from(IC_2),

$$\theta_2 \log\ q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right) - t_2$$

$$\geq \theta_2 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right) - t_1$$

$$\geq \theta_1 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right) - t_1$$

If (IR_1), is inactive, i.e., $\theta_1 \log q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right) - t_1 > 0$

then (IR_2) is inactive, i.e. $\theta_2 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right) - t_2 > 0$

In this case, if the a service provider raises price from t_1 to t_2, then it can increase its profit.

So, (IR_1) is active at the optimum

$$t_1 = \theta_2 \log\ q_1 - \frac{1}{N}\log\left(\frac{K}{K - n_1q_1 - n_2q_2}\right).$$

② Since $\theta_2 \geq \theta_1$, $q_1 \geq 0$, and (IR_1) is active, if (IC_2) is inactive, then

In this case, if the a service provider raises price t_2 subject to (IC_2), then it can increase its profit.

So, (IC_2) is active, i.e. $t_2 - t_1 = \theta_2(\log q_2 - \log q_1)$.

As a result, we get $t_2 = t_1 + \theta_2(\log q_2 - \log q_1)$.

③ Sum (IC_1) and (IC_2), then

$$\left\{\theta_1 \log q_1 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_1\right\}$$

$$+ \left\{\theta_2 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_2\right\}$$

$$\geq \left\{\theta_1 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right)\right.$$

$$\left. + \left\{\theta_2 \log q_1 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_1\right\}\right.$$

That is, $\theta_2 \log q_2 - \theta_2 \log q_1 \geq \theta_1 \log q_2 - \theta_1 \log q_1 > 0$, since $\theta_2 \geq \theta_1$.
Therefore, $\theta_2 (\log q_2 - \log q_1) > 0$.
Consequently, $q_2 \geq q_1$.

④ From ②, $t_2 - t_1 = \theta_2 (\log q_2 - \log q_1)$

And from ③, $\theta_2 \log q_2 - \theta_2 \log q_1 \geq \theta_1 \log q_2 - \theta_1 \log q_1 > 0$.
Therefore, we get $t_2 - t_1 = \theta_2 (\log q_2 - \log q_1) \geq \theta_1 (\log q_2 - \log q_1) > 0$.

$$t_2 - t_1 = \theta_2 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right)$$

$$-\theta_2 \log q_1 \left(\frac{K}{K-n_1q_1-n_2q_2}\right)$$

$$\geq \theta_1 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right)$$

$$-\theta_1 \log q_1 + \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) > 0$$

So, $\theta_1 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_1 \geq \theta_1 \log q_2$

$$+\frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_2 > 0$$

Therefore, we can neglect (IC_1).

From ①, $\theta_2 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_2$

$$\geq \theta_2 \log q_2 + \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_1$$

$$> \theta_1 \log q_1 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_2 = 0$$

So, $\theta_2 \log q_2 - \frac{1}{N}\log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) - t_2 \geq 0.$

Therefore, we can neglect (IC_2).

Proposition 1

Model (7) can be rewritten as max

$$\left\{N\theta_1 \log q_1 - \log\left(\frac{K}{K-n_1q_1-n_2q_2}\right) + n_2\theta_2(\log q_2 - \log q_2\right\}.$$

From FOC $\frac{\partial \Pi}{\partial q_1} = \frac{N\theta_1}{q_1} - \frac{n_1}{K-n_1q_1-n_2q_2} - \frac{n_2\theta_2}{q_1} = 0$

and $\frac{\partial \Pi}{\partial q_2} = -\frac{n_1}{K-n_1q_1-n_2q_2} + \frac{n_2\theta_2}{q_2} = 0,$

we get $q_2 = \frac{n_1\theta_2}{N\theta_1 - n_2\theta_2} q_1$, where $\frac{n_1\theta_2}{N\theta_1 - n_2\theta_2} \geq 1$, since $\theta_2 \geq \theta_1$.

Thus, $q_1^* = \dfrac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)}\ K.$

Since $\dfrac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)} \le 1,\ N\theta_1 - n_2\theta_2 \ge 0,$ we get $\dfrac{(1-n_1)N\theta_1 - n_1}{n_2} \le \theta_2 \le \dfrac{N\theta_1}{n_2}$

As a result, $p_2^* = \dfrac{\theta_2}{1+N\theta_1}\ K.$ Since $\dfrac{\theta_2}{1+N\theta_1} \le 1,\ \theta_2 \le 1+N\theta_1.$

Since $(1+N\theta_1) - \dfrac{N\theta_1}{n_2} = 1 + \dfrac{(n_2-1)N\theta_1}{n_2} \ge 0,\quad \dfrac{(1-n_1)N\theta_1 - n_1}{n_2} \le \theta_2 \le \dfrac{N\theta_1}{n_2}.$

Therefore, we get $p_1^* = \dfrac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)}\ K,\ p_2^* = \dfrac{\theta_2}{1+N\theta_1}\ K.$

Since a service provider should set

$$t_1^* = \theta_2 \log\ q_1^* - \frac{1}{N}\log\ (1+N\theta_1) > 0,\ q_1^* > (1+N\theta_1)^{\frac{1}{N\theta_1}}.$$

From $q_1^* = \dfrac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)}\ K$ and $q_1^* > (1+N\theta_1)^{\frac{1}{N\theta_1}},$

$$\theta_2 < \frac{N\theta_1}{n_2} - \frac{n_1}{n_1 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}.$$

Consequently, we get $\dfrac{(1-n_1)N\theta_1 - n_1}{n_2} \le \theta_2 < \dfrac{N\theta_1}{n_2} - \dfrac{n_1}{n_2 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}$

Therefore,

$$Q^* = n_1 q_1^* + n_2 q_2^* = \frac{N\theta_1}{1+N\theta_1}K,\ \text{if}\ \frac{(1-n_1)N\theta_1 - n_1}{n_2} \le \theta_2$$

$$< \frac{N\theta_1}{n_2} - \frac{n_1}{n_1 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}},$$

$$Q^* = n_2 q_2^* = \frac{n_2\theta_2}{1+N\theta_1}K, \text{ if } \theta_2 < \frac{(1-n_1)N\theta_1 - n_1}{n_2}$$

$$\text{or } \theta_2 \geq \frac{N\theta_1}{n_2} - \frac{n_1}{n_2 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}$$

Similarly, from model (8), $q^{**} = \dfrac{\theta_2}{1+N\theta_1} K$ and $Q^{**} = \dfrac{N\theta_1}{1+N\theta_1}K$

From model (9),

$$q_1^{***} = \left(\frac{N\theta_1 - n_2\theta_2}{n_1\theta_1}\right)\frac{K}{N}, \ q_2^{***} = \left(\frac{\theta_2}{\theta_1}\right)\frac{K}{N}, \text{ and } Q^{***} = K$$

Since $\dfrac{N\theta_1}{1+N\theta_1} \leq 1$, we get $Q^* \leq Q^{**} \leq Q^{***}$.

Proposition 2

Since $q_1^* < q^{**}$, we get $\dfrac{q_2^{**}}{q_1^*} > \dfrac{q^{**}}{q_1^*}$.

Then, $\log\left(\dfrac{q_2^*}{q_1^*}\right) \geq \dfrac{N\theta_1}{n_2\theta_2}\log\left(\dfrac{q^{**}}{q_1^*}\right) \geq \log\left(\dfrac{q^{**}}{q_1^*}\right)$,

which leads to $\theta_2(\log q_2^* - \log q_1^*) \geq \dfrac{N\theta_1}{n_2}(\log q^{**} - \log q_1^*)$.

Therefore, we can show

$$N\theta_1(\log q_2^* - \log q^{**}) + n_2\theta_2(\log q_2^* - \log q_1^*) \geq 0.$$

$$\Pi^* - \Pi^{**} = \left\{N\theta_1 \log q_1^* - \log(1+N\theta_1) + n_2\theta_2(\log q_2^* - \log q_1^*)\right\}$$

$$- \left\{N\theta_1 \log q^{**} - \log(1+N\theta_1)\right\}$$

$$= N\theta_1(\log q_2^* - \log q^*) + n_2\theta_2(\log q_2^* - \log q_1^*) \geq 0.$$

That is, $\Pi^* \geq \Pi^{**}$.

Proposition 3

From the proof of proposition 1, we get the optimal quantities and prices as following:

$$q_1^* = \frac{N\theta_1 - n_2\theta_2}{n_1(1+N\theta_1)}K, \quad q_2^* = \frac{\theta_2}{1+N\theta_1}K$$

$$t_1^* = \theta_1 \log\ q_1^* - \frac{1}{N}\log(1+N\theta_1), \quad t_2^* = t_1^* + \theta_2(\log\ q_2^* - \log\ q_1^*)$$

where $q_1^* = t_1^* = 0$ if $\theta_2 < \dfrac{(1-n)N\theta_1 - n_1}{n_2}$ or

$$\theta_2 \geq \frac{N\theta_1}{n_2} - \frac{n_1}{n_2 K}(1+N\theta_1)^{1+\frac{1}{N\theta_1}}$$

Therefore, we get $t_2^* - t_1^* = \theta_2(\log\ q_2^* - \log\ q_1^*)$,

where log q_2^* – log q_1^* > since $q_2^* > q_1^*$ as shown in ③.
As a result, we can show $t_2^* > t_1^*$.

Proposition 4

From the proof of proposition 3, we can get

$$p_2^* - p_1^* = \frac{t_2^*}{q_2^*} - \frac{t_1^*}{q_1^*} = \frac{1}{q_1^* q_2^*}(q_1^* t_2^* - q_2^* t_1^*)$$

$$= \frac{1}{q_1^* q_2^*}\left[q_1^*\left\{t_1^* + \theta_2(\log\ q_2^* - \log\ q_1^*)\right\} - q_1^* t_1^*\right]$$

$$= \frac{1}{q_2^*}\left\{\theta_2(\log\ q_2^* - \log\ q_1^*) - (q_2^* - q_1^*)\frac{t_1^*}{q_1^*}\right\},$$

where $\theta_2(\log\ q_2^* - \log\ q_1^*) - (q_2^* - q_1^*)\dfrac{t_1^*}{q_1^*}$

means $\theta_2(\log\ q_2^* - \log\ q_1^*) - (q_2^* - q_1^*)\ \ p_1^*$.

Therefore, $p_2^* < p_1^*$ if $\theta_2(\log\ q_2^* - \log\ q_1^*) < (q_2^* - \log\ q_1^*)\ \ p_1^*$.

$p_2^* > p_1^*$ if $\theta_2(\log\ q_2^* - \log\ q_1^*) > (q_2^* - \log\ q_1^*)\ \ p_1^*$.

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