# 무선인지 시스템에서의 부분 채널 정보만을 이용하는 전력 할당 기법

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# Power Allocation for Cognitive Radio Systems with Partial Channel State Information

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## 요 약

본 논문은 OFDM 기반 무선인지 시스템에서 부시스템의 채널 상태는 완벽하게 알지만 주시스템까지의 채널 상태는 부분적으로만 안다고 가정하였을 때의 전력 할당을 다룬다. 우리는 각 부채널 별로 최대 간섭량이 정해져 있는 주시스템과 모든 부채널의 평균 간섭량이 얼마 이하까지는 견딜 수 있는 주시스템 두 가지를 고려하였다. 이 두 가지 주시스템에 대해서 기존의 전송 전력 제약조건에 추가적으로 간섭 전력 초과 확률을 제약조건을 가지는 용량 최대화 문제를 세웠고, 이를 풀기 위한 최적 및 준최적의 전력 할당 방식을 제안하였으며 다양한 시뮬레이션을 통하여 그 성능을 검증하였다.

## 1. Introduction

Cognitive radio (CR) is a highly promising technology to solve the spectrum insufficiency problem. In spectrum sharing based CR networks, where a secondary (unlicensed) system coexists with a primary (licensed) system, a fundamental design problem is how to maximize the capacity of the secondary user (SU) while ensuring the quality of service (QoS) of the primary user (PU).

In the CR setting, optimal power allocation algorithms have been developed for orthogonal frequency division multiplexing (OFDM) systems [3] and for multiple input multiple output (MIMO) systems [4]. In order to keep the interference at the PU receiver (PU-Rx) below a desired level, these papers assume that the SU transmitter (SU-Tx) is fully aware of the channel from the SU-Tx to the PU-Rx. However, compared to the *intra-system* channel state information (CSI) between the SU-Tx and the SU receiver (SU-Rx), which is relatively easy to obtain, it would be difficult or even infeasible for the SU-Tx to obtain the perfect *inter-system* CSI because the primary and secondary systems are usually loosely coupled. Even if they are tightly coupled, it may not be easy due to the large feedback overhead. Therefore, assuming only partial CSI between the SU and the PU seems to be a reasonable approach. In this paper, we focus on a problem of maximizing capacity in OFDM-based CR systems, where the SU-Tx has *perfect intra-system CSI* and *partial inter-system CSI*. We also investigate how much more capacity can be obtained if SU is operating in band with a more sophisticated PU instead of a dumb PU.

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## 2. System Model

Consider an OFDM-based CR network where both PU and SU share the same spectrum resource with $N$ subchannels in bandwidth $B$. Denote by $\mathcal{N} = \{1, 2, \cdots, N\}$ the set of all subchannels. The signal model of an SU can be represented as $\mathbf{y} = \mathbf{D}_{\mathbf{h_2}}\mathbf{x} + \mathbf{z}$, where vectors $\mathbf{y}$, $\mathbf{x}$ and $\mathbf{z}$ are the received, transmitted signals and noise, respectively; $\mathbf{D}_{\mathbf{h_2}}$ is a diagonal matrix with diagonal elements $\mathbf{h_2} = [h_{21}, \cdots, h_{2N}]^T$, which is a channel response from the SU-Tx to the SU-Rx. The channel response from the SU-Tx to the PU-Rx is denoted by a vector $\mathbf{h_1} = [h_{11}, \cdots, h_{1N}]^T$.

Suppose that the SU-Tx has perfect CSI for its own link $\mathbf{h_2}$. In other words, it knows instantaneous channel gains $g_{2n} = |h_{2n}|^2$ for all subchannels $n \in \mathcal{N}$. However, due to the lack of inter-system cooperation, the PU intermittently informs the SU-Tx of only partial CSI about $\mathbf{h_1}$. Based on the assumption that a subchannelization with sufficient interleaving depth is applied, we use an uncorrelated fading channel model [6]. Therefore, in this case, the $\mathbf{h_1}$ is a zero-mean complex Gaussian random vector and the channel gains $g_{1n} = |h_{1n}|^2$ for all $n \in \mathcal{N}$ are independent and identically distributed (i.i.d.) exponential random variables with mean $\lambda_1$. The partial CSI includes this average channel gain $\lambda_1$, and we further assume that the channel is time-varying and frequency-selective but the mean remains unchanged until the next feedback.

## 3. PROBLEM DEFINITION

Our objective is to determine an optimal transmit power allocation vector $\mathbf{p} = [p_1, \cdots, p_N]^T$ of SU-Tx such that the capacity of the SU is maximized while the QoS of the PU is guaranteed by keeping an outage probability within a target level $\epsilon$. We define the outage probability $P_{out}(\cdot)$ as the probability that the interference power to the PU is greater than a threshold, i.e., interference-temperature $I_{max,n}$ or $I_{max}$. Motivated by these considerations, we mathematically formulate two types of optimization problems.

The first problem assumes that the PU is a dumb (peak interference-power tolerable) system that can tolerate a certain amount of peak interference at each subchannel. Thus, in this problem, we attempt to find an optimal power allocation vector $\mathbf{p}$ for maximizing the capacity under a total transmit-power constraint and a *peak* interference-power outage constraint.

$$\text{[P1]:} \max_{\mathbf{p}\geq 0} \sum_{n\in\mathcal{N}} B\log_2\left(1+\frac{g_{2n}p_n}{N_0 B}\right) \tag{1}$$

$$\text{subject to } \textstyle\sum_{n\in\mathcal{N}} p_n \leq P_{max}, \tag{2}$$

$$P_{out}(\mathbf{p}) = Pr[g_{1n}p_n > I_{max,n}] \leq \epsilon, \forall n, \tag{3}$$

where $N_0$ is the noise power spectral density and $P_{max}$ is the maximal transmit power of the SU; $I_{max,n}$ is the peak interference temperature threshold that the PU can tolerate at subchannel $n$, which may differ from subchannel to subchannel.

In the second problem, we assume that the PU operates in a more sophisticated system rather than the dumb system. The PU has an average interference-power tolerable capability so that it can tolerate the interference from the SU as long as the average of interference over all subchannels is within a certain threshold. The rationale behind this averaging assumption is that even though there is large interference in some subchannels, small interference in the other subchannels can compensate for the performance of PU in an average sense. Thus, in this problem, we try to find an optimal power allocation vector $\mathbf{p}$ for maximizing the capacity under a total transmit-power constraint and a *average* interference-power outage constraint.

$$\text{[P2]:} \max_{\mathbf{p}\geq 0} \sum_{n\in\mathcal{N}} B\log_2\left(1+\frac{g_{2n}p_n}{N_0 B}\right) \tag{4}$$

$$\text{subject to } \textstyle\sum_{n\in\mathcal{N}} p_n \leq P_{max}, \tag{5}$$

$$P_{out}(\mathbf{p}) = Pr\Big[\frac{1}{N}\sum_{n\in\mathcal{N}} g_{1n}p_n > I_{max}\Big] \leq \epsilon, \tag{6}$$

where $I_{max}$ is the average interference temperature threshold that the PU can tolerate over all subchannels.

## 4. POWER ALLOCATION UNDER PARTIAL CSI

### 4.1. Peak Interference-Power tolerable PU

The first problem [P1] is the same as the classical water-filling [7] except the peak interference-power outage constraint (3). Since $g_{1n}$ is assumed to follow an exponential distribution, we can rewrite this constraint (3) as follows:

$$p_n \leq I_{max,n}/F_E^{-1}(1-\epsilon), \quad \forall n \in \mathcal{N}, \tag{7}$$

where $F_E^{-1}(\cdot)$ is the inverse cumulative density function (CDF) of exponential distribution with the mean $\lambda_1$. Note that $F_E^{-1}(1-\epsilon)$ can be interpreted as an *effective channel gain*. The constraint (7), which limits the maximum allocable transmit power on each subchannel $n$, is additionally introduced into the classical water-filling problem. Therefore, we can obtain the optimal power allocation algorithm for [P1], so called *capped water-filling*:

Algorithm for [P1]

$$p_n = [1/\mu - N_0B/g_{2n}]_0^{I_{max,n}/F_E^{-1}(1-\epsilon)}, \ \forall n \in \mathcal{N}, \tag{8}$$

where $[z]_a^b \doteq \min[\max[a,z], b]$; $\mu$ is a non-negative Lagrange multiplier associated with (2) and is chosen such that $\sum_n p_n(\mu) = \min\left[P_{max}, \sum_n I_{max,n}/F_E^{-1}(1-\epsilon)\right]$.

### 4.2. Average Interference-Power tolerable PU

To deal with the second problem [P2], let us introduce random variables $X_n = p_n g_{1n}$ for all $n$, which are independently exponential distributed with mean $p_n\lambda_1$, and $X$ denotes the sum of these random variables. Then, the outage constraint (6) can be rewritten as:

$$Pr\left[X = \textstyle\sum_n X_n > N \cdot I_{max}\right] \leq \epsilon. \tag{9}$$

To further examine the constraint (9), it is necessary to know the distribution of $X$. However, in general, it is hard to explicitly determine the distribution of $X$. Thus, we use the Gaussian approximation[1] based on the *Lyapunov's central limit theorem (CLT)* [8]. For a large number of subchannels $N$, $X$ can be approximated to a normally distributed random variable with mean $m \simeq \sum_n p_n\lambda_1$ and variance $\sigma^2 \simeq \sum_n (p_n\lambda_1)^2$. Thus, we can rewrite the constraint (9) as:

$$P_{out}(\mathbf{p}) = \frac{1}{2}erfc\left(\frac{NI_{max}-m}{\sqrt{2}\sigma}\right) \leq \epsilon, \tag{10}$$

where $erfc(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-t^2}dt$.

If a power allocation is given, then we can easily check whether it satisfies the outage constraint (10) or not. Unfortunately, however, it is challenging to solve the problem [P2] simultaneously considering both constraints (5) and (10) because (10) has a very complicated form. Alternatively, we develop a suboptimal power allocation algorithm, which repeatedly (however, it is very fast because we require only a few iterations via binary search.) solves a subproblem having only a transmit-power constraint using the classical water-filling algorithm and then adjusts the available transmit power $\overline{P}$ until the desired outage probability $\epsilon$ is achieved.

*Lemma 1:* The $P_{out}(\mathbf{p})$ is a *strictly increasing* function of the available transmit power $\overline{P}$ if the conventional WATERFILLING($\overline{P}$) is applied, i.e., $p_n = [1/\mu - N_0B/g_{2n}]^+, \forall n$, where $\mu$ satisfies $\sum_n p_n = \overline{P}$.

[1] In order to apply the Lyapunov's CLT, the following condition should be satisfied: $\lim_{N\to\infty} \left[\sum_{n=1}^N E[(X_n - m_n)^3]\right]^{1/3} \Big/ \left[\sum_{n=1}^N \sigma_n^2\right]^{1/2} = 0$, where $m_n$ and $\sigma_n^2$ are mean and variance of the exponential distributed random variable $X_n$, respectively. We omit the proof due to the paper length limitation.

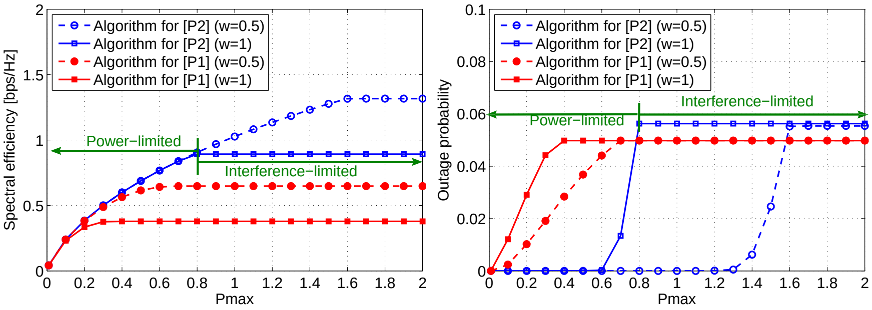


Fig. 1. The performance of proposed algorithms: $N$=128 and $\epsilon$=0.05

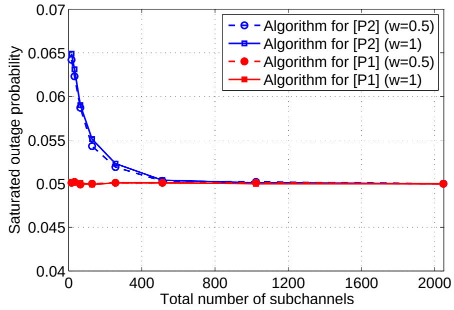


Fig. 2. Saturated outage probability

**Algorithm for [P2]**

1: Initialize: $\overline{P} = P_{max}$ and $\mathbf{p} = \text{WATERFILLING}(P_{max})$.
   if $P_{out}(\mathbf{p}) > \epsilon + \delta$, then $[a, b] \leftarrow [0, P_{max}]$,
   else, go to Finish.
   where $\delta$ is a small positive constant controlling an accuracy.

2: Repeat: $\overline{P} = (a + b)/2$ and $\mathbf{p} = \text{WATERFILLING}(\overline{P})$.
   if $P_{out}(\mathbf{p}) > \epsilon + \delta$, then $[a, b] \leftarrow [a, \overline{P}]$,
   else if $P_{out}(\mathbf{p}) < \epsilon - \delta$, then $[a, b] \leftarrow [\overline{P}, b]$,
   else, go to Finish.

3: Finish: $\mathbf{p}$ is a suboptimal power allocation.

## 5. SIMULATIONS RESULTS

Without loss of generality, the total noise power over the spectrum $(N_0 B) \cdot N$ is set to be one and the interference-temperature thresholds are adapted to the level of noise, i.e., $I_{max,n} = I_{max} = 1/N$ for all $n$. The channel gains $\{g_{1n}\}$ and $\{g_{2n}\}$ are i.i.d. exponential random variables with mean $\lambda_1$ and $\lambda_2$, respectively. We obtain numerical results based on $10^5$ randomly generated channel realizations.

We examine the performance of power allocation algorithms by choosing $N = 128$ and $\epsilon = 0.05$. Fig. 1 shows the spectral efficiency for the SU and the outage probability for the PU with respect to the maximal transmit power for different combinations of ratio $w = \lambda_1/\lambda_2$ (we fix $\lambda_2 = 1$ and vary $\lambda_1$). In the low $P_{max}$ regime, the spectral efficiency increases as the available power increases. On the other hand, when $P_{max}$ is greater than a certain turning point, the spectral efficiency does not further increase because the interference-power outage constraints becomes dominant. We indicate the boundary of power-limited and interference-limited regimes in the case of [P1] and $w = 1$ in the middle of figures.

Reducing the ratio $w$ increases the spectral efficiency due to loose interference-power outage constraints because the PU goes far away from the SU. It is important to highlight that the SU can always obtain the higher spectral efficiency in [P2] than [P1], e.g., more than two times in terms of the saturated performance. This is because the more sophisticated PU instead of the dumb one gives additional freedom in power allocation to the SU. We may confirm this argument by comparing the interference-power outage constraint of [P2] with that of [P1]. Since the average interference-power outage constraint (6) is looser than the peak interference-power outage constraint (3) at the same interference-temperature $I_{max,n} = I_{max}$, more flexible power allocation is possible.

In the power-limited regime, the outage probability is much lower than the target $\epsilon = 0.05$. If we keep increasing $P_{max}$ until the interference-limited regime, then the outage probability is saturated to the target. The optimal algorithm for [P1] always achieves the exact target requirement, while the suboptimal algorithm for [P2] exhibits a small deviation from the target value due to Gaussian approximation error.

We then investigate the relationship between the total number of subchannels and Gaussian approximation error. As you can see in Fig. 2, the saturated outage probability sticks to the target outage level as the number of subchannels $N$ increases. In other words, the approximation error asymptotically goes to zero. However, if the system does not have the sufficient number of subchannels, a suitable margin on the target error probability is required to make the system robust.

## 6. CONCLUSION

In this paper, we proposed power allocation algorithms for OFDM-based CR systems where the intra-system CSI is perfectly known and inter-system CSI is partially known, i.e., only channel mean is available. We also observed that more than two times of capacity (in terms of the saturated performance) can be obtained if the SU is operating in band with a more sophisticated PU instead of a dumb PU. Extension to more general channel models that includes correlation or feedback delay is a subject for future work.

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