

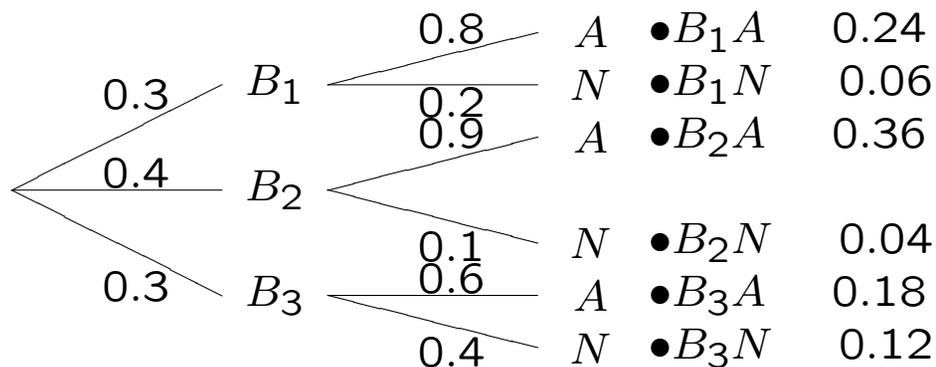
Section 2.1

Tree Diagrams

Example 2.1 Problem

For the resistors of Example 1.16, we used A to denote the event that a randomly chosen resistor is “within 50Ω of the nominal value.” This could mean “acceptable.” We use the notation N (“not acceptable”) for the complement of A . The experiment of testing a resistor can be viewed as a two-step procedure. First we identify which machine (B_1 , B_2 , or B_3) produced the resistor. Second, we find out if the resistor is acceptable. Draw a tree for this sequential experiment. What is the probability of choosing a resistor from machine B_2 that is not acceptable?

Example 2.1 Solution



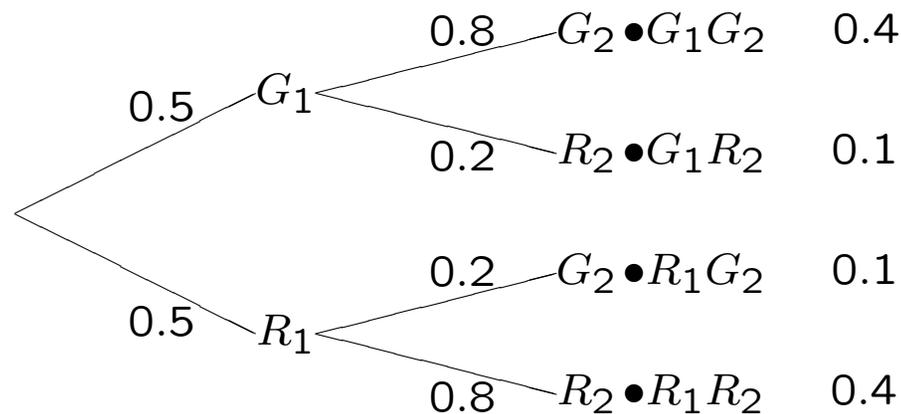
This two-step procedure is shown in the tree on the left. To use the tree to find the probability of the event B_2N , a nonacceptable resistor from machine B_2 , we start at the left and find that the prob-

ability of reaching B_2 is $P[B_2] = 0.4$. We then move to the right to B_2N and multiply $P[B_2]$ by $P[N|B_2] = 0.1$ to obtain $P[B_2N] = (0.4)(0.1) = 0.04$.

Example 2.2 Problem

Traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green, what is the probability $P[G_2]$ that the second light is green? Also, what is $P[W]$, the probability that you wait for at least one of the first two lights? Lastly, what is $P[G_1|R_2]$, the conditional probability of a green first light given a red second light?

Example 2.2 Solution



The tree for the two-light experiment is shown on the left. The probability that the second light is green is

$$\begin{aligned} P[G_2] &= P[G_1G_2] + P[R_1G_2] \\ &= 0.4 + 0.1 = 0.5. \end{aligned} \quad (2.1)$$

The event W that you wait for at least one light is the event that at least one light is red.

$$W = \{R_1G_2 \cup G_1R_2 \cup R_1R_2\}. \quad (2.2)$$

The probability that you wait for at least one light is

$$P[W] = P[R_1G_2] + P[G_1R_2] + P[R_1R_2] = 0.1 + 0.1 + 0.4 = 0.6. \quad (2.3)$$

[Continued]

Example 2.2 Solution

(Continued 2)

An alternative way to the same answer is to observe that W is also the complement of the event that both lights are green. Thus,

$$P[W] = P[(G_1G_2)^c] = 1 - P[G_1G_2] = 0.6. \quad (2.4)$$

To find $P[G_1|R_2]$, we need $P[R_2] = 1 - P[G_2] = 0.5$. Since $P[G_1R_2] = 0.1$, the conditional probability that you have a green first light given a red second light is

$$P[G_1|R_2] = \frac{P[G_1R_2]}{P[R_2]} = \frac{0.1}{0.5} = 0.2. \quad (2.5)$$

Example 2.3 Problem

In the Monty Hall game, a new car is hidden behind one of three closed doors while a goat is hidden behind each of the other two doors. Your goal is to select the door that hides the car. You make a preliminary selection and then a final selection. The game proceeds as follows:

- You select a door.
- The host, Monty Hall (who knows where the car is hidden), opens one of the two doors you didn't select to reveal a goat.
- Monty then asks you if you would like to switch your selection to the other unopened door.
- After you make your choice (either staying with your original door, or switching doors), Monty reveals the prize behind your chosen door.

To maximize your probability $P[C]$ of winning the car, *is switching to the other door either (a) a good idea, (b) a bad idea or (c) makes no difference?*

Example 2.3 Solution

To solve this problem, we will consider the “switch” and “do not switch” policies separately. That is, we will construct two different tree diagrams: The first describes what happens if you switch doors while the second describes what happens if you do not switch.

First we describe what is the same no matter what policy you follow. Suppose the doors are numbered 1, 2, and 3. Let H_i denote the event that the car is hidden behind door i . Also, let's assume you first choose door 1. (Whatever door you do choose, that door can be labeled door 1 and it would not change your probability of winning.) Now let R_i denote the event that Monty opens door i that hides a goat. If the car is behind door 1 Monty can choose to open door 2 or door 3 because both hide goats. He chooses door 2 or door 3 by flipping a fair coin. If the car is behind door 2, Monty opens door 3 and if the car is behind door 3, Monty opens door 2. Let C denote the event that you win the car and G the event that you win a goat. After Monty opens one of the doors, you decide whether to change your choice or stay with your choice of door 1. Finally, Monty opens the door of your final choice, either door 1 or the door you switched to.

[Continued]

Example 2.3 Solution

(Continued 2)

The tree diagram in Figure 2.1(a) applies to the situation in which you change your choice. From this tree we learn that when the car is behind door 1 (event H_1) and Monty opens door 2 (event R_2), you switch to door 3 and then Monty opens door 3 to reveal a goat (event G). On the other hand, if the car is behind door 2, Monty reveals the goat behind door 3 and you switch to door 2 and win the car. Similarly, if the car is behind door 3, Monty reveals the goat behind door 2 and you switch to door 3 and win the car. For always switch, we see that

$$P[C] = P[H_2R_3C] + P[H_3R_2C] = 2/3. \quad (2.6)$$

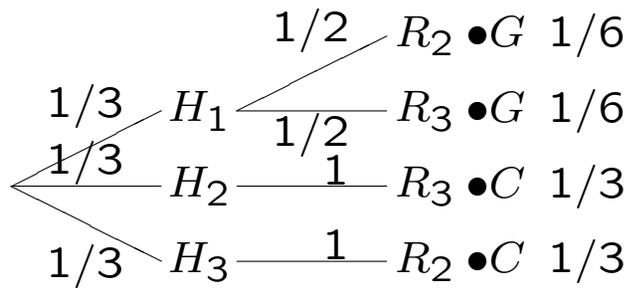
If you do not switch, the tree is shown in Figure 2.1(b). In this tree, when the car is behind door 1 (event H_1) and Monty opens door 2 (event R_2), you stay with door 1 and then Monty opens door 1 to reveal the car. On the other hand, if the car is behind door 2, Monty will open door 3 to reveal the goat. Since your final choice was door 1, Monty opens door 1 to reveal the goat. For do not switch,

$$P[C] = P[H_1R_2C] + P[H_1R_3C] = 1/3.$$

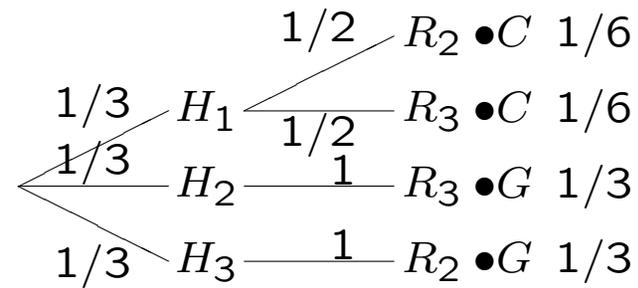
Thus switching is better; if you don't switch, you win the car only if you initially guessed the location of the car correctly, an event that occurs with probability 1/3. If you switch, you win the car when your initial guess was wrong, an event with probability 2/3.

Note that the two trees look largely the same because the key step where you make a choice is somewhat hidden because it is implied by the first two branches followed in the tree.

Figure 2.1



(a) Switch



(b) Do Not Switch

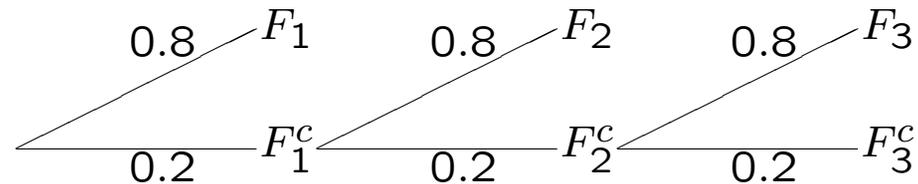
Tree Diagrams for Monty Hall

Quiz 2.1

In a cellular phone system, a mobile phone must be paged to receive a phone call. However, paging attempts don't always succeed because the mobile phone may not receive the paging signal clearly. Consequently, the system will page a phone up to three times before giving up. If the results of all paging attempts are independent and a single paging attempt succeeds with probability 0.8, sketch a probability tree for this experiment and find the probability $P[F]$ that the phone receives the paging signal clearly.

Quiz 2.1 Solution

Let F_i denote the event that the user is found on page i . The tree for the experiment is



The user is found unless all three paging attempts fail. Thus the probability the user is found is

$$P[F] = 1 - P[F_1^c F_2^c F_3^c] = 1 - (0.2)^3 = 0.992. \quad (1)$$

Section 2.2

Counting Methods

Example 2.4 Problem

Choose 7 cards at random from a deck of 52 different cards. Display the cards in the order in which you choose them. How many different sequences of cards are possible?

Example 2.4 Solution

The procedure consists of seven subexperiments. In each subexperiment, the observation is the identity of one card. The first subexperiment has 52 possible outcomes corresponding to the 52 cards that could be drawn. For each outcome of the first subexperiment, the second subexperiment has 51 possible outcomes corresponding to the 51 remaining cards. Therefore there are 52×51 outcomes of the first two subexperiments. The total number of outcomes of the seven subexperiments is

$$52 \times 51 \times \cdots \times 46 = 674,274,182,400. \quad (2.7)$$

Fundamental Principle of

Theorem 2.1 **Counting**

An experiment consists of two subexperiments. If one subexperiment has k outcomes and the other subexperiment has n outcomes, then the experiment has nk outcomes.

Example 2.5

There are two subexperiments. The first subexperiment is “Flip a coin and observe either heads H or tails T .” The second subexperiment is “Roll a six-sided die and observe the number of spots.” It has six outcomes, $1, 2, \dots, 6$. The experiment, “Flip a coin and roll a die,” has $2 \times 6 = 12$ outcomes:

$(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),$
 $(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6).$

Theorem 2.2

The number of k -permutations of n distinguishable objects is

$$(n)_k = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}.$$

Example 2.6

Suppose there are four objects, A , B , C , and D , and we define an experiment in which the procedure is to choose two objects without replacement, arrange them in alphabetical order, and observe the result. In this case, to observe AD we could choose A first or D first or both A and D simultaneously. The possible outcomes of the experiment are AB , AC , AD , BC , BD , and CD .

Example 2.7

Suppose there are four objects, A , B , C , and D , and we define an experiment in which the procedure is to choose two objects without replacement and observe the result. The 12 possible outcomes of the experiment are AB , AC , AD , BA , BC , BD , CA , CB , CD , DA , DB , and DC .

Theorem 2.3

The number of ways to choose k objects out of n distinguishable objects is

$$\binom{n}{k} = \frac{(n)_k}{k!} = \frac{n!}{k!(n-k)!}.$$

Definition 2.1 n choose k

For an integer $n \geq 0$, we define

$$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!} & k = 0, 1, \dots, n, \\ 0 & \text{otherwise.} \end{cases}$$

Example 2.8

- The number of combinations of seven cards chosen from a deck of 52 cards is

$$\binom{52}{7} = \frac{52 \times 51 \times \cdots \times 46}{2 \times 3 \times \cdots \times 7} = 133,784,560, \quad (2.11)$$

which is the number of 7-combinations of 52 objects. By contrast, we found in Example 2.4 674,274,182,400 7-permutations of 52 objects. (The ratio is $7! = 5040$).

- There are 11 players on a basketball team. The starting lineup consists of five players. There are $\binom{11}{5} = 462$ possible starting lineups.
- There are $\binom{120}{60} \approx 10^{36}$ ways of dividing 120 students enrolled in a probability course into two sections with 60 students in each section.
- A baseball team has 15 field players and ten pitchers. Each field player can take any of the eight nonpitching positions. The starting lineup consists of one pitcher and eight field players. Therefore, the number of possible starting lineups is $N = \binom{10}{1} \binom{15}{8} = 64,350$. For each choice of starting lineup, the manager must submit to the umpire a batting order for the 9 starters. The number of possible batting orders is $N \times 9! = 23,351,328,000$ since there are N ways to choose the 9 starters, and for each choice of 9 starters, there are $9! = 362,880$ possible batting orders.

Example 2.9 Problem

There are four queens in a deck of 52 cards. You are given seven cards at random from the deck. What is the probability that you have no queens?

Example 2.9 Solution

Consider an experiment in which the procedure is to select seven cards at random from a set of 52 cards and the observation is to determine if there are one or more queens in the selection. The sample space contains $H = \binom{52}{7}$ possible combinations of seven cards, each with probability $1/H$. There are $H_{NQ} = \binom{52-4}{7}$ combinations with no queens. The probability of receiving no queens is the ratio of the number of outcomes with no queens to the number of outcomes in the sample space. $H_{NQ}/H = 0.5504$.

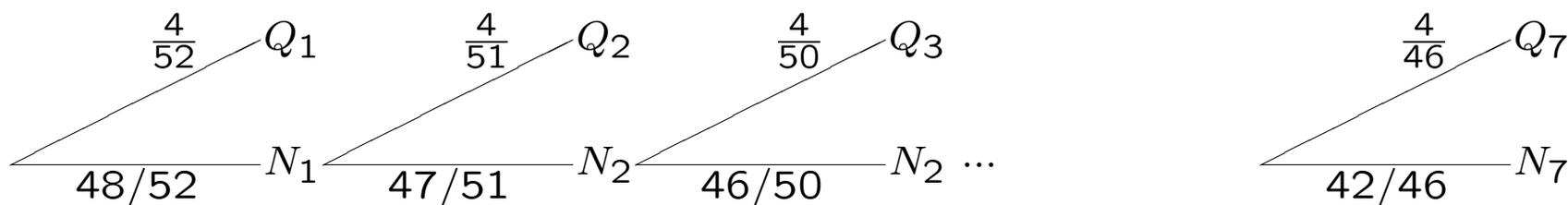
Another way of analyzing this experiment is to consider it as a sequence of seven subexperiments. The first subexperiment consists of selecting a card at random and observing whether it is a queen. If it is a queen, an outcome with probability $4/52$ (because there are 52 outcomes in the sample space and four of them are in the event {queen}), stop looking for queens.

[Continued]

Example 2.9 Solution

(Continued 2)

Otherwise, with probability $48/52$, select another card from the remaining 51 cards and observe whether it is a queen. This outcome of this subexperiment has probability $4/51$. If the second card is not a queen, an outcome with probability $47/51$, continue until you select a queen or you have seven cards with no queen. Using Q_i and N_i to indicate a “Queen” or “No queen” on subexperiment i , the tree for this experiment is



The probability of the event N_7 that no queen is received in your seven cards is the product of the probabilities of the branches leading to N_7 :

$$(48/52) \times (47/51) \cdots \times (42/46) = 0.5504. \quad (2.12)$$

Example 2.10 Problem

There are four queens in a deck of 52 cards. You are given seven cards at random from the deck. After receiving each card you return it to the deck and receive another card at random. Observe whether you have not received any queens among the seven cards you were given. What is the probability that you have received no queens?

Example 2.10 Solution

The sample space contains 52^7 outcomes. There are 48^7 outcomes with no queens. The ratio is $(48/52)^7 = 0.5710$, the probability of receiving no queens. If this experiment is considered as a sequence of seven subexperiments, the tree looks the same as the tree in Example 2.9, except that all the horizontal branches have probability $48/52$ and all the diagonal branches have probability $4/52$.

Theorem 2.4

Given m distinguishable objects, there are m^n ways to choose with replacement an ordered sample of n objects.

Example 2.11

There are $2^{10} = 1024$ binary sequences of length 10.

Example 2.12

A chip fabrication facility produces microprocessors. Each microprocessor is tested to determine whether it runs reliably at an acceptable clock speed. A subexperiment to test a microprocessor has sample space $S_{\text{sub}} = \{0, 1\}$ to indicate whether the test was a failure (0) or a success (1). For test i , we record $x_i = 0$ or $x_i = 1$ to indicate the result. In testing four microprocessors, the observation sequence, $x_1x_2x_3x_4$, is one of 16 possible outcomes:

$$S = \left\{ \begin{array}{l} 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, \\ 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111 \end{array} \right\}.$$

Theorem 2.5

For n repetitions of a subexperiment with sample space $S_{\text{sub}} = \{s_0, \dots, s_{m-1}\}$, the sample space S of the sequential experiment has m^n outcomes.

Example 2.13

There are ten students in a probability class; each earns a grade $s \in S_{\text{sub}} = \{A, B, C, F\}$. We use the notation x_i to denote the grade of the i th student. For example, the grades for the class could be

$$x_1 x_2 \cdots x_{10} = CBBACFBACF \quad (2.13)$$

The sample space S of possible sequences contains $4^{10} = 1,048,576$ outcomes.

Example 2.14 Problem

For five subexperiments with sample space $S_{\text{sub}} = \{0, 1\}$, what is the number of observation sequences in which 0 appears $n_0 = 2$ times and 1 appears $n_1 = 3$ times?

Example 2.14 Solution

The 10 five-letter words with 0 appearing twice and 1 appearing three times are:

$\{00111, 01011, 01101, 01110, 10011, 10101, 10110, 11001, 11010, 11100\}$.

Theorem 2.6

The number of observation sequences for n subexperiments with sample space $S = \{0, 1\}$ with 0 appearing n_0 times and 1 appearing $n_1 = n - n_0$ times is $\binom{n}{n_1}$.

Theorem 2.7

For n repetitions of a subexperiment with sample space $S = \{s_0, \dots, s_{m-1}\}$, the number of length $n = n_0 + \dots + n_{m-1}$ observation sequences with s_i appearing n_i times is

$$\binom{n}{n_0, \dots, n_{m-1}} = \frac{n!}{n_0! n_1! \dots n_{m-1}!}.$$

Proof: Theorem 2.7

Let $M = \binom{n}{n_0, \dots, n_{m-1}}$. Start with n empty slots and perform the following sequence of subexperiments:

Subexperiment	Procedure
0	Label n_0 slots as s_0 .
1	Label n_1 slots as s_1 .
\vdots	\vdots
$m - 1$	Label the remaining n_{m-1} slots as s_{m-1} .

There are $\binom{n}{n_0}$ ways to perform subexperiment 0. After n_0 slots have been labeled, there are $\binom{n-n_0}{n_1}$ ways to perform subexperiment 1. After subexperiment $j - 1$, $n_0 + \dots + n_{j-1}$ slots have already been filled, leaving $\binom{n-(n_0+\dots+n_{j-1})}{n_j}$ ways to perform subexperiment j . From the fundamental counting principle,

$$\begin{aligned}
 M &= \binom{n}{n_0} \binom{n-n_0}{n_1} \binom{n-n_0-n_1}{n_2} \dots \binom{n-n_0-\dots-n_{m-2}}{n_{m-1}} \\
 &= \frac{n!}{(n-n_0)!n_0!} \frac{(n-n_0)!}{(n-n_0-n_1)!n_1!} \dots \frac{(n-n_0-\dots-n_{m-2})!}{(n-n_0-\dots-n_{m-1})!n_{m-1}!}.
 \end{aligned} \tag{2.14}$$

Canceling the common factors, we obtain the formula of the theorem.

Definition 2.2 Multinomial Coefficient

For an integer $n \geq 0$, we define

$$\binom{n}{n_0, \dots, n_{m-1}} = \begin{cases} \frac{n!}{n_0! n_1! \cdots n_{m-1}!} & \begin{array}{l} n_0 + \cdots + n_{m-1} = n; \\ n_i \in \{0, 1, \dots, n\}, i = 0, 1, \dots, m-1, \end{array} \\ 0 & \text{otherwise.} \end{cases}$$

Example 2.15 Problem

In Example 2.13, the professor uses a *curve* in determining student grades. When there are ten students in a probability class, the professor always issues two grades of A, three grades of B, three grades of C and two grades of F. How many different ways can the professor assign grades to the ten students?

Example 2.15 Solution

In Example 2.13, we determine that with four possible grades there are $4^{10} = 1,048,576$ ways of assigning grades to ten students. However, now we are limited to choosing $n_0 = 2$ students to receive an A , $n_1 = 3$ students to receive a B , $n_2 = 3$ students to receive a C and $n_3 = 4$ students to receive an F . The number of ways that fit the curve is the multinomial coefficient

$$\binom{n}{n_0, n_1, n_2, n_3} = \binom{10}{2, 3, 3, 2} = \frac{10!}{2!3!3!2!} = 25,200. \quad (2.16)$$

Quiz 2.2

Consider a binary code with 4 bits (0 or 1) in each code word. An example of a code word is 0110.

- (a) How many different code words are there?
- (b) How many code words have exactly two zeroes?
- (c) How many code words begin with a zero?
- (d) In a constant-ratio binary code, each code word has N bits. In every word, M of the N bits are 1 and the other $N - M$ bits are 0. How many different code words are in the code with $N = 8$ and $M = 3$?

Quiz 2.2 Solution

- (a) We can view choosing each bit in the code word as a subexperiment. Each subexperiment has two possible outcomes: 0 and 1. Thus by the fundamental principle of counting, there are $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible code words.
- (b) An experiment that can yield all possible code words with two zeroes is to choose which 2 bits (out of 4 bits) will be zero. The other two bits then must be ones. There are $\binom{4}{2} = 6$ ways to do this. Hence, there are six code words with exactly two zeroes. For this problem, it is also possible to simply enumerate the six code words:

1100, 1010, 1001,
0101, 0110, 0011.

- (c) When the first bit must be a zero, then the first subexperiment of choosing the first bit has only one outcome. For each of the next three bits, we have two choices. In this case, there are $1 \times 2 \times 2 \times 2 = 8$ ways of choosing a code word.
- (d) For the constant ratio code, we can specify a code word by choosing M of the bits to be ones. The other $N - M$ bits will be zeroes. The number of ways of choosing such a code word is $\binom{N}{M}$. For $N = 8$ and $M = 3$, there are $\binom{8}{3} = 56$ code words.

Section 2.3

Independent Trials

Example 2.16 Problem

What is the probability $P[E_{2,3}]$ of two failures and three successes in five independent trials with success probability p .

Example 2.16 Solution

To find $P[E_{2,3}]$, we observe that the outcomes with three successes in five trials are 11100, 11010, 11001, 10110, 10101, 10011, 01110, 01101, 01011, and 00111. We note that the probability of each outcome is a product of five probabilities, each related to one subexperiment. In outcomes with three successes, three of the probabilities are p and the other two are $1 - p$. Therefore each outcome with three successes has probability $(1 - p)^2 p^3$.

From Theorem 2.6, we know that the number of such sequences is $\binom{5}{3}$. To find $P[E_{2,3}]$, we add up the probabilities associated with the 10 outcomes with 3 successes, yielding

$$P[E_{2,3}] = \binom{5}{3} (1 - p)^2 p^3. \quad (2.17)$$

Theorem 2.8

The probability of n_0 failures and n_1 successes in $n = n_0 + n_1$ independent trials is

$$P [E_{n_0, n_1}] = \binom{n}{n_1} (1 - p)^{n - n_1} p^{n_1} = \binom{n}{n_0} (1 - p)^{n_0} p^{n - n_0}.$$

Example 2.17 Problem

In Example 1.16, we found that a randomly tested resistor was acceptable with probability $P[A] = 0.78$. If we randomly test 100 resistors, what is the probability of T_i , the event that i resistors test acceptable?

Example 2.17 Solution

Testing each resistor is an independent trial with a success occurring when a resistor is acceptable. Thus for $0 \leq i \leq 100$,

$$P [T_i] = \binom{100}{i} (0.78)^i (1 - 0.78)^{100-i} \quad (2.18)$$

We note that our intuition says that since 78% of the resistors are acceptable, then in testing 100 resistors, the number acceptable should be near 78. However, $P[T_{78}] \approx 0.096$, which is fairly small. This shows that although we might expect the number acceptable to be close to 78, that does not mean that the probability of exactly 78 acceptable is high.

Theorem 2.9

A subexperiment has sample space $S_{\text{sub}} = \{s_0, \dots, s_{m-1}\}$ with $P[s_i] = p_i$. For $n = n_0 + \dots + n_{m-1}$ independent trials, the probability of n_i occurrences of s_i , $i = 0, 1, \dots, m-1$, is

$$P \left[E_{n_0, \dots, n_{m-1}} \right] = \binom{n}{n_0, \dots, n_{m-1}} p_0^{n_0} \cdots p_{m-1}^{n_{m-1}}.$$

Example 2.18

A packet processed by an Internet router carries either audio information with probability $7/10$, video, with probability $2/10$, or text with probability $1/10$. Let $E_{a,v,t}$ denote the event that the router processes a audio packets, v video packets, and t text packets in a sequence of 100 packets. In this case,

$$P[E_{a,v,t}] = \binom{100}{a,v,t} \left(\frac{7}{10}\right)^a \left(\frac{2}{10}\right)^v \left(\frac{1}{10}\right)^t \quad (2.23)$$

Keep in mind that by the extended definition of the multinomial coefficient, $P[E_{a,v,t}]$ is nonzero only if $a + v + t = 100$ and a , v , and t are nonnegative integers.

Quiz 2.3

Data packets containing 100 bits are transmitted over a communication link. A transmitted bit is received in error (either a 0 sent is mistaken for a 1, or a 1 sent is mistaken for a 0) with probability $\epsilon = 0.01$, independent of the correctness of any other bit. The packet has been coded in such a way that if three or fewer bits are received in error, then those bits can be corrected. If more than three bits are received in error, then the packet is decoded with errors.

- (a) Let $E_{k,100-k}$ denote the event that a received packet has k bits in error and $100 - k$ correctly decoded bits. What is $P[E_{k,100-k}]$ for $k = 0, 1, 2, 3$?
- (b) Let C denote the event that a packet is decoded correctly. What is $P[C]$?

Quiz 2.3 Solution

- (a) In this problem, k bits received in error is the same as k failures in 100 trials. The failure probability is $\epsilon = 1 - p$ and the success probability is $1 - \epsilon = p$. That is, the probability of k bits in error and $100 - k$ correctly received bits is

$$P [E_{k,100-k}] = \binom{100}{k} \epsilon^k (1 - \epsilon)^{100-k}. \quad (1)$$

For $\epsilon = 0.01$,

$$P [E_{0,100}] = (1 - \epsilon)^{100} = (0.99)^{100} = 0.3660. \quad (2)$$

$$P [E_{1,99}] = 100(0.01)(0.99)^{99} = 0.3700. \quad (3)$$

$$P [E_{2,98}] = 4950(0.01)^2(0.99)^{98} = 0.1849. \quad (4)$$

$$P [E_{3,97}] = 161,700(0.01)^3(0.99)^{97} = 0.0610. \quad (5)$$

- (b) The probability a packet is decoded correctly is just

$$P [C] = P [E_{0,100}] + P [E_{1,99}] + P [E_{2,98}] + P [E_{3,97}] = 0.9819. \quad (6)$$

Section 2.4

Matlab

Figure 2.2

Y =											
Columns 1 through 12											
47	52	48	46	54	48	47	48	59	44	49	48
Columns 13 through 24											
42	52	40	40	47	48	48	48	53	49	45	61
Columns 25 through 36											
60	59	49	47	49	45	48	51	48	53	52	53
Columns 37 through 48											
56	54	60	53	52	51	58	47	50	48	44	49
Columns 49 through 60											
50	46	52	50	51	51	57	50	49	56	44	56

The simulation output of 60 repeated experiments of 100 coin flips.

Example 2.19 Problem

Using Matlab, perform 60 experiments. In each experiment, flip a coin 100 times and record the number of heads in a vector \mathbf{Y} such that the j th element Y_j is the number of heads in subexperiment j .

Example 2.19 Solution

```
>> X=rand(100,60)<0.5;  
>> Y=sum(X,1)
```

The Matlab code for this task appears on the left.

The 100×60 matrix \mathbf{X} has i, j th element $X(i, j)=0$ (tails) or $X(i, j)=1$ (heads) to indicate the result of flip i of subexperiment j . Since Y sums X across the first dimension, $Y(j)$ is the number of heads in the j th subexperiment. Each $Y(j)$ is between 0 and 100 and generally in the neighborhood of 50. The output of a sample run is shown in Figure 2.2.

Example 2.20 Problem

In testing a microprocessor all four grades, 1, 2, 3 and 4, have probability 0.25, independent of any other microprocessor. Simulate the testing of 100 microprocessors. Your output should be a 4×1 vector \mathbf{X} such that X_i is the number of grade i microprocessors.

Example 2.20 Solution

```
%chiptest.m  
G=ceil(4*rand(1,100));  
T=1:4;  
X=hist(G,T);
```

The first line generates a row vector G of random grades for 100 microprocessors. The possible test scores are in the vector T . Lastly, $X=\text{hist}(G,T)$ returns a histogram vector X such that $X(j)$ counts the number of elements $G(i)$ that equal $T(j)$.

Note that “`help hist`” will show the variety of ways that the `hist` function can be called. Moreover, $X=\text{hist}(G,T)$ does more than just count the number of elements of G that equal each element of T . In particular, `hist(G,T)` creates bins centered around each $T(j)$ and counts the number of elements of G that fall into each bin.

Quiz 2.4

The flip of a thick coin yields heads with probability 0.4, tails with probability 0.5, or lands on its edge with probability 0.1. Simulate 100 thick coin flips. Your output should be a 3×1 vector \mathbf{X} such that X_1 , X_2 , and X_3 are the number of occurrences of heads, tails, and edge.

Quiz 2.4 Solution

For a Matlab simulation, we first generate a vector R of 100 random numbers. Second, we generate vector X as a function of R to represent the 3 possible outcomes of a flip. That is, $X(i)=1$ if flip i was heads, $X(i)=2$ if flip i was tails, and $X(i)=3$ if flip i landed on the edge. The matlab code is

```
R=rand(1,100);
X=(R<= 0.4) ...
  + (2*(R>0.4).*(R<=0.9)) ...
  + (3*(R>0.9));
Y=hist(X,1:3)
```

To see how this works, we note there are three cases:

- If $R(i) \leq 0.4$, then $X(i)=1$.
- If $0.4 < R(i)$ and $R(i) \leq 0.9$, then $X(i)=2$.
- If $0.9 < R(i)$, then $X(i)=3$.

These three cases will have probabilities 0.4, 0.5 and 0.1. Lastly, we use the `hist` function to count how many occurrences of each possible value of $X(i)$.